道路舗装の補修同期化のための簡便的ルール
A Simplified Rule of Pavement Repair Synchronization

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複数の連続した道路舗装区間の補修を検討する場合，近接する複数の舗装区間を単一の規制において補修すること（補修同期化）の便益（規模の経済性）を考慮することが望ましい．その際，現実的な規模の道路舗装区間で劣化を考慮しながら最適規制・補修区間の厳密解を求めることは，計算負荷が膨大となることから現実的でない．そこで，本研究では，簡便的ルールによる道路舗装区間の規制・補修区間決定手法を提案する．数値計算事例を通じて，i）小規模な道路舗装区間において提案手法により厳密解に近い解が得られること，ii）提案手法は厳密解の計算が不可能な大規模な道路舗装区間にも適用できること，を実証的に示す．

Key Words: *network-level optimization, simplified rule, repair policy, work zone, deterioration, life cycle cost, road asset management*

1. Introduction

To determine pavement sections to repair and work zone locations on a corridor system consisting of multiple adjacent road sections, we should consider the benefits of synchronized repairs on neighboring sections within a single work zone to produce economies of scale. For example in Fig. 1, Case A requires two units fixed repair/work zone cost for two work zones while Case B requires only one for the continuous work zone. Lethan et al.[1] have developed a method to determine the optimal work zone for large-scale networks at a single time point, but in view of long-term management of pavement system, it is desirable to consider the stochastic pavement deterioration process.

Exact optimal repair and work zone policies considering deterioration process is computationally intractable to find because the solution space of optimal repair and work zone policies for a real-scale road pavement system might be too large. In this study, a method to find close-to-optimal repair and work zone policies by a time-consistent simplified rule is proposed. Case study shows that the proposed method can find close-to-optimal policies for small-scale road pavement systems and might also be applied to real-scale road pavement systems where the optimal solution requires too much computational time to find.

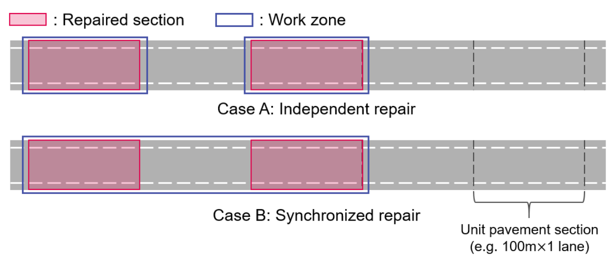

Fig. 1 Economies of scale in repair

2. Road pavement system model

(1) Deterioration and repair of system

A pavement system is composed of N linearly connected, continuous and homogeneous pavement sections with section IDs $\{1, 2, \cdots, N\}$. T is the total number of inspections planned to conduct along the finite planning horizon. The time point t indicates tth inspection timing, where interval of any two neighboring inspection timing is d. Condition state (CS) of pavement is classified into M discrete ratings. 1 is the best condition state and M is the worst condition. $s_n^-(t)$ is condition state of section n at t before repair and $\boldsymbol{s}^-(t) \equiv [s_n^-(t)]$ is the CS vector indicates inspected state of the entire system. $s_n^+(t)$ is condition state of section n at t after repair and $\boldsymbol{s}^+(t) \equiv [s_n^+(t)]$ is the CS vector indicates state of the entire system after repair. The state space of the condition state vector can be expressed as $\boldsymbol{s} \in \mathcal{S} = \mathcal{M}^N$.

Deterioration procedure of the entire system is modeled as a Markov process. The Markov transition probability is expressed as $\boldsymbol{P}_{\boldsymbol{s}^*,\boldsymbol{s}^{**}} = \mathrm{Prob}[\boldsymbol{s}^-(t) = \boldsymbol{s}^{**} | \boldsymbol{s}^+(t-1) = \boldsymbol{s}^*]$.

Repair process can be conducted after every inspection but work zone is needed to conduct repair for a pavement section. Two binary decision variables $\delta_{t,n} \in \{0, 1\}$ and $\xi_{t,n} \in \{0, 1\}$ are defined to indicate the repair policy and work zone policy for section n at t: Section n is repaired when $\delta_{t,n} = 1$ and not repaired when $\delta_{t,n} = 0$; Section n is in work zone when $\xi_{t,n} = 1$ and not in work zone when $\xi_{t,n} = 0$. A repair and work zone policy for system can be defined as $\boldsymbol{\delta}_t \equiv [\delta_{t,n}]$ and $\boldsymbol{\xi}_t \equiv [\xi_{t,n}]$. Sections with worst conditions (i.e. $s = M$) must be repaired, which means $\delta_{t,n} = 1$ if $s_n^-(t) = M$. Sections to repair must be selected as work zone, which means $\delta_{t,n} \leq \xi_{t,n}$.

(2) Determination of repair and work zone policies

Objective of the management is to minimize the life cycle cost (LCC) which comprised of the three cost factors:

- Variable repair cost: $\alpha \sum_{n=1}^{N} \delta_{t,n}$
- Variable work zone cost: $\beta \sum_{n=1}^{N} \xi_{t,n}$
- Fixed repair/work zone cost: $\gamma q_t(\boldsymbol{\xi}_t)$

, where $q_t(\boldsymbol{\xi}_t)$ is number of continuous work zones at t and is a function of $\boldsymbol{\xi}_t$. The expected discounted value of the future life cycle cost of the road pavement system at t,

$\Psi_t(\boldsymbol{\delta}_t, \boldsymbol{\xi}_t | \boldsymbol{s}^-(t))$, can be described as:

$$
\begin{aligned}
&\Psi_t(\boldsymbol{\delta}_t, \boldsymbol{\xi}_t | \boldsymbol{s}^-(t)) \\
=&\alpha \sum_{n=1}^{N} \delta_{t,n} + \beta \sum_{n=1}^{N} \xi_{t,n} + \gamma q_t(\boldsymbol{\xi}_t) \\
&+ \frac{1}{(1+\rho)^d} \sum_{\boldsymbol{s}^-(t+1) \in \mathcal{S}} \boldsymbol{P}_{\boldsymbol{s}^+(t), \boldsymbol{s}^-(t+1)} \Psi_{t+1}(\boldsymbol{\delta}_{t+1}, \boldsymbol{\xi}_{t+1} | \boldsymbol{s}^-(t+1))
\end{aligned}
\tag{1}
$$

, where ρ is a discount rate.

The optimal repair and work zone policies at t with given $\boldsymbol{s}^-(t)$, $(\boldsymbol{\delta}_t^*(\boldsymbol{s}^-(t)), \boldsymbol{\xi}_t^*(\boldsymbol{s}^-(t)))$, can be found by the following Bellman equation:

$$
\begin{aligned}
&V_t(\boldsymbol{s}^-(t)) \\
=&\alpha \sum_{n=1}^{N} \delta_{t,n} + \beta \sum_{n=1}^{N} \xi_{t,n} + \gamma q_t(\boldsymbol{\xi}_t) \\
&+ \frac{1}{(1+\rho)^d} \sum_{\boldsymbol{s}^-(t+1) \in \mathcal{S}} \boldsymbol{P}_{\boldsymbol{s}^+(t), \boldsymbol{s}^-(t+1)} V_{t+1}(\boldsymbol{s}^-(t+1))
\end{aligned}
\tag{2}
$$

, where $V_t(\boldsymbol{s}^-(t))$ is the optimal value function of $\Psi_t(\boldsymbol{\delta}_t, \boldsymbol{\xi}_t | \boldsymbol{s}^-(t))$, defined as $\Psi_t(\boldsymbol{\delta}_t^*(\boldsymbol{s}^-(t)),\ \boldsymbol{\xi}_t^*(\boldsymbol{s}^-(t)) | \boldsymbol{s}^-(t))$. This is a typical Markov decision process (MDP)[2]. The entire optimal repair and work zone policy becomes a set of TM^N policies. As a result, the state space of the repair policy becomes exponentially large as scale of the problem becomes large. When number of sections in the system N is relatively small, optimal repair and work zone policies. can be found by backward recursion[2], but when N is large as real scale, it is intractable to calculate optimal policies.

To overcome the combinatorial explosion in the optimization problem to determine repair and work zone policies, we propose a method to use a time-consistent simplified rule:

" A primary section and secondary sections nearby should be repaired in a single work zone. The primary sections are defined as the sections with condition state (CS) above the primary risk control level X at an inspection timing. The secondary sections are defined as the sections with CS between the primary risk control level X and the second risk control level Y, which are located within the search distance Z near a certain detected primary section at the inspection timing. The elongation of a work zone including a primary section is automatically defined based on the identified secondary sections nearby. "

Based on the rule, $\delta_{t,n}$ and $\xi_{t,n}$ are uniquely determined with given (X, Y, Z) and $\boldsymbol{s}^-(t)$, and they are denoted by $\delta_{t,n}(\boldsymbol{s}^-(t), X, Y, Z)$ and $\xi_{t,n}(\boldsymbol{s}^-(t), X, Y, Z)$. The optimal simplified rule (X^*, Y^*, Z^*) can be given by:

$$
\begin{aligned}
&(X^*, Y^*, Z^*) \\
=& \mathop{\arg\min}_{(X,Y,Z) \in \mathcal{X} \times \mathcal{Y} \times \mathcal{Z}} \sum_{t=1}^{T} \frac{1}{(1+\rho)^{(t-1)d}} E_{\boldsymbol{s}^-(t)} \Big[\alpha \sum_{n=1}^{N} \delta_{t,n}(\boldsymbol{s}^-(t), X, Y, Z) \\
&+ \beta \sum_{n=1}^{N} \xi_{t,n}(\boldsymbol{s}^-(t), X, Y, Z) + \gamma q_t(\boldsymbol{\xi}_t(\boldsymbol{s}^-(t), X, Y, Z)) \Big]
\end{aligned}
\tag{3}
$$

, where the symbol "$\times$" indicates the Cartesian product, and $E_{\boldsymbol{s}^-(t)}[\cdot]$ stands for the expected value of the input with respect to the random vectors $\boldsymbol{s}^-(t)$. As X, Y and Z are independent of t, the value of the objective function in Equation (3) can be calculated with a given set of X, Y, and Z using the Monte Carlo simulation by generating CSs randomly. Also, the cardinality of possible simplified rules is $M(M-1)L/2$, which enables us to find the optimal solution by calculating the values of the objective function for all possible sets of X, Y, and Z. Compared to TM^N of the exact solution, the proposed rule-based method can significantly reduce the complexity to $M(M-1)L/2$.

Table. 1 Case study in small-scale system ($N = 5$)

	Exact solution	Proposed method
Computational time	7488 seconds	46 seconds
Expected life cycle cost	92.40 [m.u.]	94.00 [m.u.]
Repair and work zone policy	Indescribable	$X^* = 4, Y^* = 3, Z^* = 3$

Table. 2 Case study in large-scale system ($N = 100$)

	Exact solution for every subsystem	Proposed method
Computational time	7488 seconds	447 seconds
Expected life cycle cost	1847.97 [m.u.]	1785.90 [m.u.]
Repair and work zone policy	Indescribable	$X^* = 4, Y^* = 3, Z^* = 3$

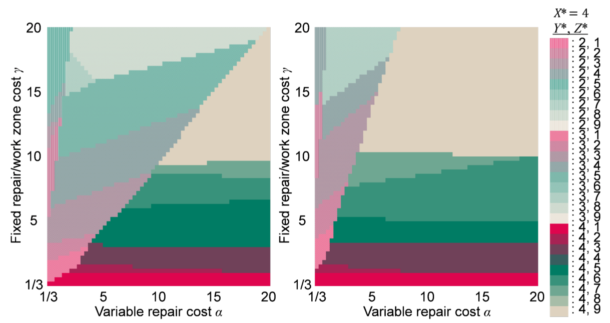

Fig. 2 Sensitivity analysis

3. Numerical study

In terms of small-scale pavement system ($N = 5$), where the optimal policy can be found, the proposed method can find a close-to-optimal policy as in Table. 1.

In terms of large-scale pavement system ($N = 100$), the proposed method can find a policy with less LCC than that to apply the optimal policy for small-scale system ($N = 5$) to divided subsystems of the entire system as in Table. 2.

Sensitivity analysis of (X^*, Y^*, Z^*) about costs is conducted by changing unit variable repair cost α and fixed repair/work zone cost γ and result is shown in Fig. 2. Two graphs are cases of different deterioration processes and it is shown that a close-to-optimal policy differs according to the condition (i.e. unit cost and deterioration process) and therefore it is necessary to find close-to-optimal repair and work zone policies case by case.

4. Conclusion

This study proposed a method to find close-to-optimal repair and work zone policies for road pavement systems by optimizing a simplified rule. The proposed method can be applied to determination of repair and work zone policies for large-scale road pavement systems where the optimal policy is computationally hard to find, and therefore more precise life cycle cost assessment for road pavement systems becomes possible. The proposed method might also be applied to determination of repair policies for other infrastructure systems with economies of scale.

Reference

1) Lethanh, N., Adey, B. T. and Burkhalter, M.: Determining an optimal set of work zones on large infrastructure networks in a GIS framework, *Journal of Infrastructure Systems*, Vol.24, Issue 1, 04017048, 2017.

2) Bellman, R.: A Markovian decision process, *Journal of Mathematics and Mechanics*, Vol.6, No.5, pp.679-684, 1957.

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