管路管理のための水道配水システムのシミュレーション
Water Distribution System Simulation for Pipeline Management

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Pipelines in water distribution system are deteriorating constantly. Most of the past studies focused on suggesting a suitable maintenance strategy considering a preset budget. However, there is a lack of methodology for setting suitable budget to ensure and improve the system performance in long-run. Deterioration causes low water availability at requirement points and also increases water loss; therefore, budget in maintenance must consider these negative effects. In this study, a methodology to set suitable budget in order to develop long-term pipeline management is developed. An optimization model based on improvement of system performance in terms of equity (water availability improvement) and efficiency (water loss reduction) is proposed. From the result, the study found the relationship between critical availability (minimum case) and replacement cost of pipelines. This relationship may help in making appropriate decisions concerning on allocation of reasonable budget in maintenance.

Key Words: *Water availability, Water requirement, Hydraulic simulation, Simulated annealing*

1. INTRODUCTION

Water authority in developing countries is facing challenges in apposite allocation of water in a scenario of supply and demand gap for coming years. Poor quality of infrastructure adds further stress in the target to reduce this gap. Among all components of a Water Distribution System (WDS), buried pipelines are subject to suitable conditions for rapid deterioration process; therefore, most of the studies related to maintenance in WDS are focused in pipelines. Common approaches use probabilistic methods in the selection of the best replacement/repair strategy by the analysis of the dynamic evolution of deterioration (e.g., timing and frequency of failures) for short-term decision and other applications. Feasible budget and its corresponding strategy for maintenance are complementary factors, however, usually budget for maintenance are assumed preset values, and there are a few studies concerning on setting feasible budget for maintenance, even if it is known that, this setting must be done in a planned way. Then, one question remains with no answer in this situation:

Does this preset budget ensure the equity and efficiency of distribution in the WDS in the long-term?

Proactive approach analyzes the short and long-term objectives in a dynamic system setting in order to avoid myopic decisions[4]. Additionally, suitable approach should consider not only optimal distribution of funds among the WDS components but also, it considers maximize the overall system's availability of water. To answer the question above, we analyzed the notion of availability of water in WDS, and its application in maintenance of pipelines.

Water availability is the ratio of supplied water related to required water at each point in the system. The situation of the lowest water availability in the WDS (critical availability) is an index that indicates equitable water distribution, at the same time, reveals the system's efficiency. *Efficiency* is defined through physical water loss (leakage) and *Equity* is represented by water availability in this case. This study tackles these characteristics for the proposal of proactive pipeline management in long-term.

(1) Aim and Scope

The main objective of this study is to provide practical information to water authorities for setting suitable budget for maintenance of pipelines and, give some features of potential replaceable pipelines as well. In order to do these, we propose a model to analyze the relationship between critical availability and replacement cost.

The specific objectives are:

- Introduce the effect of the water loss and the criterion of water availability in a hydraulic simulation of WDS.
- Analyze the relation between improvement of critical availability (*Equity*) and reduction of water loss (*Efficiency*) influenced by the replacement of pipelines.

2. METHODOLOGY AND FACTORS

The objective and our framework require that *water loss* and *water availability* are given as functions of WDS performance. In order to incorporate these factors, at first stage, we utilized a hydraulic simulation approach known as "Pressure Driven Demand" (PPD). In a second stage, optimization model (Maximin problem) is applied to find the highest possible situation of critical availability. Because of the optimization problem is non-linear due to hydraulic behavior of WDS, a heuristic method (Simulated Annealing) [1)] is applied into the model. Figure 1 summarizes our methodology.

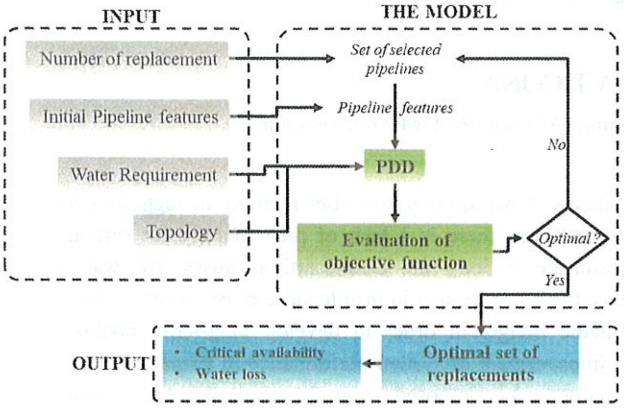

Figure 1: Flow diagram of methodology

3. THE MODEL

(1) Hydraulic Simulation: PDD

Conventional hydraulic simulation fixes water loss as percentage of water requirement, additionally they assume that systems are able to satisfy this requirement perfectly (required and supplied water are the same). These assumptions disable these approaches for our purposes and also for WDS in developing countries. To evaluate the hydraulic performance of WDS realistically, a pressure driven demand method is required. For the formulation, let's consider WDS comprises np pipelines with unknown flow, nn requirement nodes with unknown hydraulic head, no nodes with known hydraulic heads (e.g., reservoir, tanks). If H_i and H_j represents the nodal heads for nodes i and j respectively, r_k represents the coefficient of frictional loss and Q_k the flow between i and j (pipe k), the hydraulic simulation model is defined as follows:

$$\begin{bmatrix} A_{pp} & A_{pn} \\ A_{np} & 0 \end{bmatrix}\begin{bmatrix} Q \\ H \end{bmatrix} = \begin{bmatrix} -A_{po}H_o \\ -q_{act} \end{bmatrix} \Rightarrow \begin{matrix} H_i - H_j = h_{ij} = r_k \cdot Q_k{}^{\omega} & (1) \\ \sum_J Q_k - q_{actual-i} = 0 & (2) \end{matrix}$$

Vectorial form *Scalar form*

Equations (1) and (2) represent energy and mass conservation laws, respectively. Q_k belongs to a subset with J nodes which are directly connected by pipelines to node i. In the vectorial representation, $\boldsymbol{A_{pp}}$ is the diagonal matrix with head losses as elements ($r_k \cdot |Q_k|^{\omega-1}$, exponent ω depends on the equation of frictional loss), $\boldsymbol{A_{pn}}$ is the incidence matrix that represents the topology of WDS' elements; $\boldsymbol{A_{po}}$ is incidence matrix of nodes with known nodal heads, $\boldsymbol{H_o}$ is the corresponding known nodal heads vector, $\boldsymbol{A_{np}}$ is transpose matrix of $\boldsymbol{A_{pn}}$ and $\boldsymbol{q_{act}}$ is the vector of total nodal outflows. We use Hazen-Williams equation ($\omega = 1.852$) for energy losses as $r_k = l_k/\left(C_o \cdot C_H{}^k \cdot D_p{}^{k\,2.63}\right)^{1.85}$, where C_o is a constant that depends on customary unit system (0.2785 for SI units and 1.3180 for US system), $C_H{}^k$ is the roughness coefficient (For new pipelines we define $C_{H0}{}^k$ as roughness coefficient default value for each material), $D_p{}^k$ the diameter and l_k is the length of pipe k. For this model vectors $\boldsymbol{Q}$ (flow) and $\boldsymbol{H}$ (nodal heads) are state variables. The incidence matrix $\boldsymbol{A_{pn}}$ has elements $A_{pn(i,j)}$ with value *"-1"* if flow has direction from j to i, *"1"* in the opposite case and, *"0"* if j and i are not connected. A well-known methodology developed by Todini (1988) called as "Global Gradient Algorithm" (GGA) solves the nonlinear system of equations, using iterations based on Newton Raphson (NR) method. NR uses the derivatives of equations (1) and (2) in their vectorial form, from those; an iterative process is run considering a local linearization between iteration τ and $\tau + 1$ to obtain the following equation:

$$\boldsymbol{H}^{\tau+1} = \boldsymbol{A}^{-1}\boldsymbol{F} \tag{3}$$

$$\boldsymbol{Q}^{\tau+1} = \boldsymbol{Q}^{\tau} - \boldsymbol{D_{pp}}^{-1}\left(\boldsymbol{A_{pp}} \cdot \boldsymbol{Q}^{\tau} + \boldsymbol{A_{pn}} \cdot \boldsymbol{H}^{\tau+1} + \boldsymbol{A_{po}} \cdot \boldsymbol{H_o}\right) \tag{4}$$

Where:

$$\boldsymbol{A} = \boldsymbol{A_{np}} \cdot \boldsymbol{D_{pp}}^{-1} \cdot \boldsymbol{A_{pn}} + \boldsymbol{DL_{nn}} + \boldsymbol{D_{nn}} \tag{5}$$

$$\boldsymbol{F} = \boldsymbol{A_{np}} \cdot \boldsymbol{Q}^{\tau} + \boldsymbol{q_{act}} - \boldsymbol{A_{np}} \cdot \boldsymbol{D_{pp}}^{-1}\left(\boldsymbol{A_{po}} \cdot \boldsymbol{H_o} + \boldsymbol{A_{pp}} \cdot \boldsymbol{Q}^{\tau}\right) + \left(\boldsymbol{DL_{nn}} + \boldsymbol{D_{nn}}\right) \tag{6}$$

Where $\boldsymbol{D_{pp}}$ is a diagonal matrix, which elements are derivatives of $\boldsymbol{A_{pp}}$. For element k from the diagonal defined as $D_{pp}(k,k) = n \cdot r \cdot |Q_k|^{\omega-1}$. $\boldsymbol{q_{act}}$ is the sum of the actual supplied water and water loss ($\boldsymbol{q_{act}} = \boldsymbol{C_{actual}} + \boldsymbol{q_l}$) and $\boldsymbol{D_{nn}}$ and $\boldsymbol{DL_{nn}}$ their respective derivatives. These factors are explained in detail in the following sections (1a) and (2b). The iteration process ends when the sum of all flow changes from a previous iteration divided by the total flow in all pipelines is less than a convergence threshold (0.001 in our case).

(1a) Water loss

Leakage depend on many factors (e.g., material, breaks, age, etc.), from those, pressure is the factor directly related to WDS' operation and our target for the evaluation of *efficiency*. Leakage is proportional to pressure in pipes; however, outflows in PDD are evaluated through nodes. Therefore, allocation of leakage to the end nodes can be performed dividing the total leakage in proportion with the magnitude of the two nodal heads (nodal pressure), with following formulation:

$$\boldsymbol{q_l} = \tfrac{1}{2}\boldsymbol{P}\left|\boldsymbol{A_{np}}\right|\begin{bmatrix} q_{1-leak} \\ \vdots \\ q_{k-leak} \\ \vdots \\ q_{np-leak} \end{bmatrix}, \quad \boldsymbol{DL_{nn}}(i,i) = \tfrac{1}{2}\boldsymbol{P}\left|\boldsymbol{A_{np}}\right|\begin{bmatrix} \frac{dq_{1-leak}}{dP_1} \\ \vdots \\ \frac{dq_{k-leak}}{dP_k} \\ \vdots \\ \frac{dq_{np-leak}}{dP_{np}} \end{bmatrix} \tag{7}$$

For:

$$q_{k-leak} = \begin{cases} \beta_k l_k (P_k)^{\alpha_k} & if\ P_k > 0 \\ 0 & if\ P_k \leq 0 \end{cases} \text{ and } P_k = \frac{P_i + P_j}{2} \tag{8}$$

Where q_{k-leak} is the original leakage in pipelines, q_{i-leak} is the reassigned nodal leakage, l_k is the length of pipe, P_k is the average pressure in the pipe estimated as mean of pressure values at the end node i (P_i) and j (P_j) for pipe k, for $P_i = H_i - Z_i$, where Z_i represent the nodal elevation in i. β_k is called water loss coefficient and α_k represents the characteristics of discharge area of burst and breaks (set as *1.2*[3])

(1b) Water availability

Availability of water is not constant, when pressure drops to a certain level known as desired pressure, nodal required water can only be partially supplied (partial availability). There is a range of pressure in which the nodal requirement can be partially supplied (supplied water), below that, nodal requirement is completely not satisfied ($P_{minimun}$) and over the range the requirement is fully supplied ($P_{service}$) (normal supply). From this analysis we have the following equations:

$$C_{i-actual} = \begin{cases} C_i & if\ P_i > \mathrm{P}_{service} \\ C_i \cdot R_i & if\ P_{minimun} \le P_i \le P_{service} \\ 0 & if\ P_i < \mathrm{P}_{minimun} \end{cases} \quad (9)$$

$$\boldsymbol{D}_{nn}(i,i) = \begin{cases} 0 & if\ P_i > P_{service} \\ C_i \cdot \dfrac{dR_i}{dP_i} & if\ P_{minimun} \le P_i \le P_{service} \\ 0 & if\ P_i < P_{minimun} \end{cases} \quad (10)$$

Where, $C_{i-actual}$ is the supplied water node i and C_i the water requirement, for vector $[\boldsymbol{C}_{actual}{}^T = [C_{1-actual}, \dots, C_{i-actual}, \dots, C_{nn-actual}]$. The index R_i is called water availability for the evaluation of *equity*. The following equation expresses the relation of water availability and WDS's performance:

$$R_i = \left(\frac{P_i - P_{minimun}}{P_{service} - P_{minimun}}\right)^{\frac{1}{2}} \quad (11)$$

(2) Maximin Optimization model

Maximin is a decision rule which consider problems where the target is to identify the most optimal condition (maximum) from a set that consist on all possible worst cases under evaluation (minimums). In this study the minimum cases are critical availability nodes and the condition is given by the replacement of a fixed number pipelines. The objective function is enunciated as follows:

$$\max_{S_m}\left(\min_{j\in NN} R_j\,(\boldsymbol{S}_m)\right) \quad (12)$$

Subject to equation (1) and (1)

Where NN is the set of nodes in the WDS [*1,... j,..., nn*], $\boldsymbol{S}_m$ represents the vector of states related to material ($\boldsymbol{r}$) and water loss characteristics ($\boldsymbol{\beta}$) corresponding to pipelines for replacement strategy m [$\boldsymbol{S}_m \in \mathrm{F}(S) \rightarrow \{1,0\}^{Nr}$, where Nr is the given number of replacements of pipelines]. The replacement of pipelines changes their state through the variation of their corresponding roughness ($C_H{}^k \rightarrow \mathrm{C}_{H0}{}^k$) and water loss coefficients ($\beta_k \rightarrow 0$) once the replacement is carried out. To solve this optimization problem we use Simulated annealing (SA) algorithm[1].

4. APPLICATION

(1) Study case

The study has been applied on two WDS with different level of complexity. The first one is a system extracted from a simple case of an Italian city[3] (Network A – 34 pipelines and 23 nodes). The material is set to PVC for all pipes ($\mathrm{C}_{H0}{}^k = 150$), water loss is set to 37.1% ($\beta_k = 2.3532\mathrm{E}-07$ for all pipes), with condition for service level $P_{minimun} = 0$ and $P_{service} = 15$. The second system belongs to a large WDS in a Bolivian city (El Alto) synthesized to pipelines with diameter greater than 50 mm (Network B – 930 pipelines and 816 nodes with 245 centroids). The system is composed by three different types of material for $\mathrm{C}_{H0}{}^k$ values (PVC, FD, and FFD[2]). β_k has been calibrated for each pipe with micro monitoring data of pressure by the criteria of pressure error minimization using correlation coefficient [$\rho^2 = 0.667$] the results have a mean and standard deviation of $\beta_k = 8.9747E-06 \pm 2.411E-06$. We imposed the water requirement for 2036 with a total loss 27.6% (related to 56.4 million cubic meters for that year[2]). Conditions for service level are $P_{minimun} = 0$ and $P_{service} = 10$. This information and results help us to check PDD model about its robustness (Table 1) and reliability related to traditional simulation software (EPANET E.P.A., USA 2000) [$\rho^2 = 0.99$].

Table 1 Statistic of PDD (8000 evaluations for variation of β_k, $C_H{}^k$)

System	Maixmun Error		Mean Error		Mean number of iterations	CPU time (millisec.)
	Energy Balance (m)	Mass Balance (m3/s)	Energy Balance (m)	Mass Balance (m3/s)		
Network A	2.09E-14	2.16E-15	1.31E-15	2.21E-16	17	10
Network B	1.90E-01	1.43E-0.5	1.14E-02	5.34E-08	22	128

(2) Results and discussion

In Figure 2, it is plot the condition in deterioration by $\Delta\mathrm{CHW} = \mathrm{C}_{H0} - C_H$, the difference in the roughness coefficient before and after a replacement (*black lines*) to represent degradation condition (high ΔCHW) of pipes, it is also plot information of water loss (before) and availability (R_j) before and after replacement. From these results, the improvement in water availability by replacement of pipelines is visible. Some features about their selection are distinguished. The study has identified the following features: 1) Replaced pipelines are located close to the critical availability nodes, 2) Replaced pipelines are into areas with mean of high water loss (high pressure zones) and 3) Pipelines are selected considering their deterioration (e.g., age) if they have influence on critical availability nodes. In a particular case, pipelines are also selected if they have important location between water sources (e.g., Tanks, Reservoirs) and the whole system (pipeline 34' case in Network A, Figure 2–Net. A). In Figure 3, we plot pipelines in Network B (black dots) according to their value of water loss and installation date, replacements are represented by red dots for selected pipelines with high deterioration (age) and high water loss.

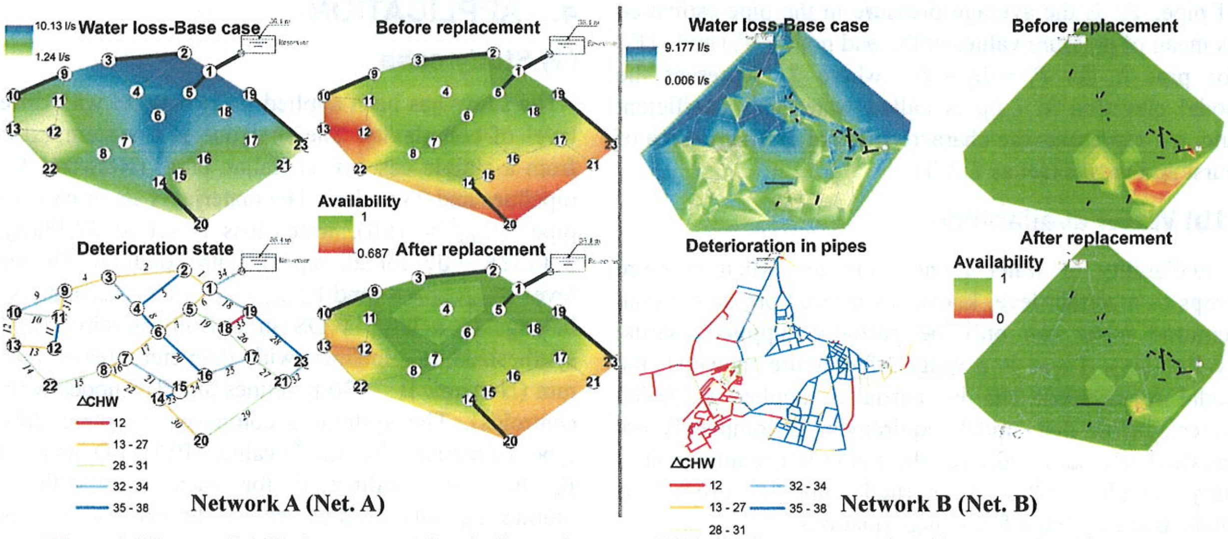

Figure 2: Water availability improvement by pipeline replacement (11% of infrastructure replaced in Net. A, 3.5% in Net. B)

From this result, we can show the features about feasible set of replaced pipelines for the improvement of efficiency and equity (Figure 2).

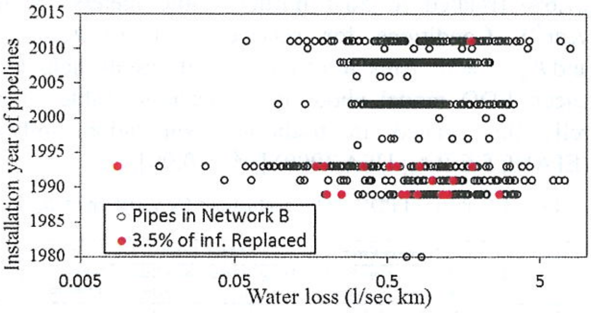

Figure 3: Distribution of replacement according to water loss and age

The relationship between critical availability and replacement cost (related to total pipeline length of system) was obtained after evaluate different number of replacement in feasible interval of improvement. The relationship behaves linear for simpler systems (Network A) and as exponential function for complex ones (Network B).

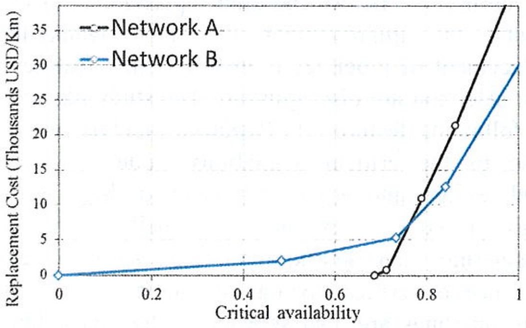

Figure 4: Relationship of critical availability and replacement cost

These happen due to size of systems. Smaller system does not have enough efficient pipelines to increase critical availability with small marginal cost even for low number of replacements; this rate keeps almost constantly as consequence. In the case of bigger and complex WDS, the existence of large feasible set of replaceable pipelines makes increase the marginal gradually (Figure 4)

5. CONCLUSIONS

The study has obtained the relationship between critical availability and replacement cost. The relationship behaves linear for simpler systems and as exponential function for complex ones. For latter case, marginal cost for system improvement becomes higher for ensuring larger values of critical availability. We found that, in systems with average or higher critical availability such as Network A, that the possible availability improvement is significant, as well as water loss reduction (global improvement in the system is ensured). On the other hand, in system with critical availability below the average such as Network B, local conditions (e.g. high elevation nodes) of critical availability points may have big influence in system improvement. For these types of cases, feasible replacements are located in order to increase local critical situation rather than improve global system performance, as consequence availability improvement increases with no significant water loss reduction. This relationship may help water authorities to set suitable budget for replacement of pipelines under improvement of system performance in long-term.

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