Optimal railway commuting pattern under flexible work time system[1)]

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ABSTRACT: Metropolitan railway commuting service market is a compound market, which consists of many sub-markets differentiated by departure time. While a regional monopolistic railway company allocates its resource to all sub-markets, by decision of train schedule, commuters choose their departure time considering the trade-off between congestion rate and schedule cost, without considering external economies. Equilibrium is not socially optimal, there are many possible policies to intervene the market and realize higher social welfare. Among them, we focus on the policies aiming to level the peak of demand, such as time differentiated fare system, staggered work hour system, and flexible work time system, and so on.

In order to describe the supplier's resource allocation and users' choice among sub-markets, we developed a partial equilibrium model of railway commuting service market, and developed methodology to analyze the equilibrium, as well as the solution under policy intervention. We have studied how we evaluate the economic effect of such kind of policies by using the partial equilibrium models. This paper introduces the basic idea of the equilibrium models and illustrates the typical results of the numerical analyses.

1) **KeyWords;** transportation policy, TDM, railway commuting, optimal control

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1. Introduction

In the Japanese metropolices, where motor transportation service level is very low by congestion, urban railway systems have important role for daily commuting. Because many commuters have no alternative, railway commuting seems very compulsory and rarely elastic. Urban railway companies provide commuting service in each sub-region in the metropolitan area very monopolistic way. Owing to the enlargement investments of the railway capacity and to the decreasing total commuting demand by the recession in the late decade, railway congestion level has been gradually decreased. Even though, the present congestion level is far from satisfactory level. Considering the limits for money and spaces, we cannot expect large-scale facility expansion any more. Then recently, transportation demand management (TDM) measures to change departure time distribution are focused in order to level the peak of demand.

In these 5 years, several experiments have become accomplished to estimate the effect of demand side measures, like staggered work hours. However, there is no theoretical assessments comparing the effect and feasibility of such demand side measures (TDM) to other supply-side measures like improvement of traffic scheduling, occasionally called as transport system management (TSM). In order to analyze a railway commuting market, interactions between railway traffic scheduling and departure time choice behavior must be modeled. Commuters want to ride less-congested trains, while a railway company tries to make trains congested considering the profit, these two objectives essentially contradict each other. A railway company has monopolistic power and tries to cut the number of trains. On the other hand, commuters choose their departure time based on personal utility, ignoring the external effect of his/her choice. As a result, equilibrium of the market becomes socially sub-optimal. TSM and TDM measures should not be evaluated as a substitute of facility expansion only, but also as a tool to adjust the monopolized market in order to realize higher social welfare level.

Furthermore, change of work schedule may affect the effectiveness of business activities. Even if a staggered work hours system reduces the dis-utilities in transportation market, it may decrease the outcome of the activities measured by income level. There is an essential trade off between transportation and business activity efficiency. Without consideration of such trade off, no TDM measures can get any corporation of firms and commuters.

2. Approach of the study

2.1 Related studies up to now

It was late 1960's when the people came to realize that expansion of transportation facilities can not solve the transportation congestion problem if they consider time and cost for the expansion.

Then, they began to study the distribution of commuting departure time and investigate the possibility to level the distribution.

A pioneering study utilized queuing theory to describe waiting time at one bottleneck[1)], and it was expanded for road network consists of many links[2)]. When commuters choose their departure time, they meet with trade off between travel time and schedule cost, which is related to the difference between desired time and actual departure time. At first, deterministic models, describing such trade off, were developed to investigate the existence or uniqueness of equilibrium[3)]. Remarkable development of logit models in 1980's enabled them to build stochastic model and simultaneous models including route choice as well as time selection. Most of these models divide time axis to several number of periods and formulated discrete choice models, but continuous optimal condition along time axis were also investigated by optimal control theory [4)5)].

These studies all analyzed departure time distribution of commuting along highway network with bottlenecks based on fixed performance function. However in railway system, traffic capacity is not constant along time, rather determined by the supplier's behavior of a railway company. Above studies are not directly applicable to the railway commuting.

On the other hand, railway commuting demand distribution along time has been studied from late 1980's. Ieda *et al*. developed logit model to describe the commuters' choice over the trade off among time, congestion and transfer. Based on the model, they estimated economic evaluation of several service schedule alternatives [6)], but cost structure of railway company was missed in these studies.

From more practical viewpoint, social experiments of TDM measures become reported in 1990's. Some of them report strong reluctance of firms to join the staggered work hours and flexible work hours, and explain it by negative effect on business efficiency. Business activities locate in urban area because they expect better accessibility to other activities and more effective interactions with others. Hall (1991) insisted that such effect is also existent on the time axis and called it by the term of "**temporal agglomeration economy**"[7)]. When you happen to need urgent communication with some person in other firm, and if the person is not on work, you must wait for him/her. Staggered work hours have a possibility to increase such temporal mismatches and harm the temporal agglomeration effect. The effect of temporal agglomeration effect in commuting under flexible time system was first analyzed by Henderson[8)9)]. Recently, Mun and Yonekawa(1997) analyzed firm's incentive for flexible work time system considering temporal agglomeration[10)] . That paper shares the viewpoint with our study, but based on bottleneck queuing model only applicable to automobile commuting.

We began the study by developing a partial equilibrium model to describe the interaction between railway company and commuters (Kobayashi. et al (1997))[11)]. As discussed later commuting service market is differentiated by departure time. Commuters can choose sub-market by deciding his/her

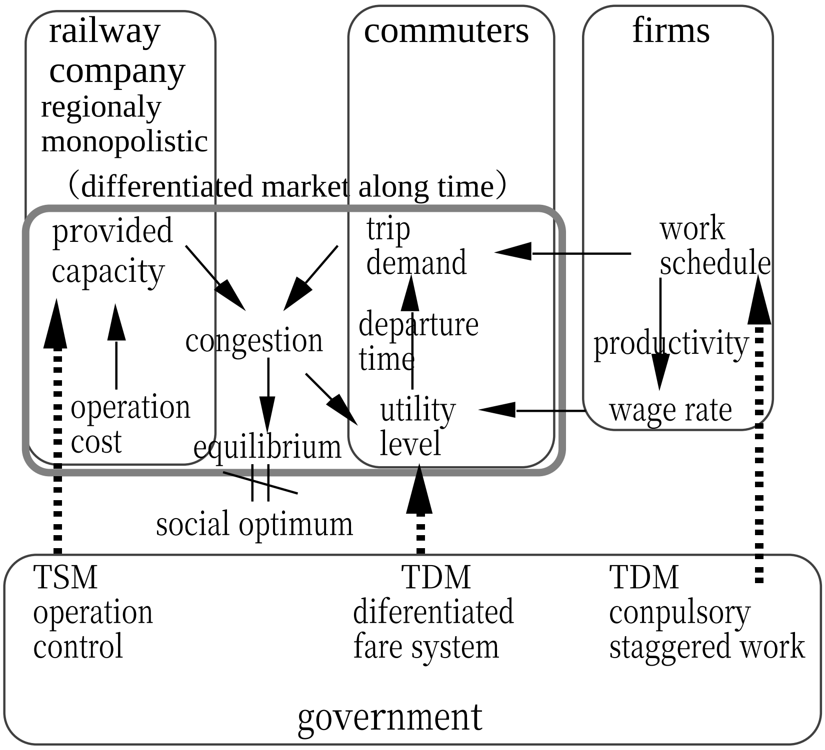

Figure 1. Structure of the railway commuting markets

departure time. In the field of urban economics, optimal control model skillfully analyzed spatially differentiated market[12)], that give a good guidance for to our studies. We have been expanding our first model to cope with the above idea, for example, in Okumura *et al*. (1998)[13)], work schedule division was introduced as exogenous parameter, that was endogenized by Okumura *et al.* (1999)[14)].

2.2 Structure of railway commuting problem

In railway commuting service market, the following four types of players interact each other; 1) railway company, 2) commuters, 3) firms and 4) government (**figure 1**).

The commuting service market is differentiated by departure time. In other words, based on the balance of supply and demand on each departure time, corresponding congestion level is determined. Congestion level can be different along time. There are many sub-markets along departure time. Commuters choose which sub-market do they enter, while the railway company, spatially monopolizing the market, can control supply level of all sub-markets.

Commuters behave as follows based on the work schedule designated by firms. They choose departure time to maximize their utility level, considering the train schedule given by the railway company. Each commuter considers the disutility he/she feels from the congestion level, but ignores the externality: if he/she rides on the train, other passenger feels worse from the congestion change given by him/her. On the other hand, the monopolistic railway company can decide the train schedule to maximize his profit. The train schedule will affect on the congestion level, and the commuters' choice again. After several cycles of feedbacks, the train schedule and congestion level will be stabilized on the equilibrium pattern.

Firms decide the working schedules. They consider temporal agglomeration effects and try to make schedule effective. If all firms have the same technology and be threatened by entrants having the same technology, the labor market becomes perfect competitive. Temporal agglomeration effect will be perfectly offset by higher wage rate, which compensates more congested commuting.

What situation will be realized under the interactions among these players? How become the congestion pattern and work schedule? Are they socially effective? These questions are our main interests.

2.3 Aim of the study

Our studies began by modeling the interaction between railway company and commuters[11)] . For simplicity, we have considered one essential unit of commuter railway, connecting CBD station and a station in housing area. Work start time of all commuters are set one point in time, in the first study.

However, in real world, there is wide variety of work schedules, decided considering each firm's situation. Peak point of work start time, such as 9:00 am for example, is not exogenously decided but endogenously created by each firms' choice. Therefore, we investigate what distribution of work start time would be realized by rational choice of firms and commuters, and how effective is that equilibrium, as well as what distribution of work start time does optimize social welfare, that is defined as aggregated commuters utility minus railway operating cost.

The former equilibrium pattern is expected in the situation where all firms introduce flexible work time system and offer performance wage system to the workers. This problem is considered as "user equilibrium" where utility level of commuters are kept same regardless the work schedule.

The latter "system optimum" situation can not be realized without policy interventions, because utility level of commuters may be different according to office arrival time, work start time and office leave time. If railway fares are perfectly differentiated along arrival time or the government levies office tax perfectly differentiated by work start time, such intervention can adjust the utility level to meet equilibrium condition and optimal working and commuting schedule can be realized. In reality, we do not have such perfect control measure. But still, the optimum solution shows the upper limit of economic evaluation of TDM and TSM policies.

3. Formulation of the model

3.1 Problem settings of our studies

We suppose one commuter railway connecting two stations, one in housing area and the other in CBD, where all of N commuters have their job. No other transportation mode cannot be used for commuting, therefore, daily demand is fixed as N demand. Commuting time for all commuters is equal to constant w, regardless to their departure time or housing location. From this assumption, we can use arrival time distribution instead of departure time distribution. All workers work for just H hours a day. Firms in the city are located at the CBD. They can specify the working schedules (work start time and work end time) for some group of workers, or introduce flexible time system and let each worker to choose his/her working schedule by him/herself, in that case, the firms offer a set of wage rates based on the productivity of the work schedules. Once a working schedule is set by a firm or worker, late coming or early leaving office are forbidden.

3.2 Specification of commuters behavior and operation cost

Utility level of representative commuter is defined as the following function of work start time: t office arrival time: t_g, and leave office time: t_r.

$$W(t, t_g, t_r) = Y(t) - R + U(t_g) + V(t_r) \quad (1)$$

$$U(t_g) = -(s(t_g))^\eta - c(T - t_g) \quad (2)$$

$$V(t_r) = -(r(t_r))^\eta - e\{t_r - (T + H)\} \quad (3)$$

where, $Y(t)$: wage for work between t and $t+H$, R: railway fare level a day, a constant regardless for t_g and t_r, $U(t_g)$: morning part of utility, $V(t_r)$: evening part of utility, T: arbitrary origin point in time, $s(t_g)$: congestion level of trains arriving CBD at t_g in the morning, $r(t_r)$: congestion level of trains departing CBD at t_r in the evening. They are positive and $s(t_g)=1$, $r(t_r)=1$ mean that number of passengers of the train equals to the number of seats. $c\ (>0)$: slope of schedule cost for early departure in the morning, while $e\ (>0)$: slope of schedule cost for late return home in the evening. In our studies, assume $c=e$ for simplicity.

Firms locating at CBD input labor to produce a *numerare* good. All firms have the same technology subject to the temporal agglomeration effect. From the free entry condition, labor market becomes perfectly competitive, then all produced value are equally divided to each workers. We formulate it by using instant production function like Henderson[9] as follows;

$$Y(t) = A\int_t^{t+H} N(\tau)^\alpha d\tau \quad (4)$$

where, A: technology level parameter, $N(t)$: number of the labor at work in the city at time t. Numbers of workers are standardized that total number of workers in the city equals to 1.

Let us define $n(t)$ as the number of workers who start work before t. $N(t)$ can be calculated by

$$N(t) = n(t) - n(t-H) \tag{5}$$

If H is long enough that there is a instance when all workers are at work, the following equation gives the wage rate.

$$Y(t) = A\left\{\int_t^{T_2} n(\tau)^\alpha d\tau + \int_{T_1}^{t} (1-n(\tau))^\alpha\, d\tau + (T_1 + H - T_2)\right\} \tag{6}$$

where, T_0: office arrival time of the first commuter, T_2+H: leave office time of the last commuter. Daily railway operating cost is considered as the following integral of instant operating cost, which is a increase function of supply rates: $u(t_g)$, $v(t_r)$ at time t_g and t_r, respectively.

$$TC \;=\; \int_{T_0}^{T_2} \zeta u(t_g)^\iota dt_g + \int_{T_1+H}^{T_3+H} \zeta v(t_r)^\iota dt_r \tag{7}$$

where, $\zeta\ (>0)$: parameter, $\iota\ (>1)$: elasticity of the instant cost functions. Let us consider that fare level is regulated to be the average cost. Because total number of commuters is standardized as 1, the average daily fare level per commuter: R becomes ,

$$R = TC \tag{8}$$

Because total demand is fixed as 1, change of railway fare does not have any effect either on railway company's behavior or on commuters' choice. That is nothing more than a transfer of income between them. Social welfare is kept constant by such transfers.

3.3 System optimum model under flexible time

When work hour length is constant as H for all commuters, the system optimum flexible time problem can be formulated as the following optimum control problem.

$$\max_{S(t),u(t),R(t),v(t),k(t)} SW = \int_{T_0}^{T_3} \left[\dot{m}(t)U(t) - \zeta u(t)^\iota + \dot{l}(t)V(t) - \zeta v(t)^\iota + \dot{n}(t)Y(t)\right] dt \tag{9}$$

$$s.t.\quad \dot{m}(t) = S(t)^{\frac{1}{\eta}}u(t) \equiv f_1(\mathbf{w}(t))(= s(t)u(t)) \quad \text{arrival rate} \tag{10}$$

$$\dot{l}(t) = R(t)^{\frac{1}{\eta}}v(t) \equiv f_2(\mathbf{w}(t))(= r(t)v(t)) \quad \text{leave rate} \tag{11}$$

$$\dot{n}(t) = k(t) \equiv f_3(\mathbf{w}(t)) \quad \text{work start rate (work end rate)} \tag{12}$$

$$\dot{Y}(t) = \begin{cases} A\left\{(1-n(t))^\alpha - n(t)^\alpha\right\}N^\alpha \\ \ if\ \ (T_1 \le t \le T_2) \cap (\dot{n}(t) \ne 0) \\ 0 \quad if\ \ (t < T_1, T_2 < t) \cup (\dot{n}(t) = 0) \end{cases}$$

$$\equiv f_4(\mathbf{w}(t)) \quad \text{change rate of wage} \tag{13}$$

$$h_1(x(t)) = -l(t) \leq 0$$ non-negative cumulative leaves (14)

$$h_2(x(t)) = l(t) - n(t) \leq 0$$ leave after ending work (15)

$$h_3(x(t)) = n(t) - m(t) \leq 0$$ arrive before starting work (16)

$$h_4(x(t)) = m(t) - 1 \leq 0$$ non-excess cumulative arrival (17)

$$m(T_0) = l(T_1) = n(T_1) = Y(T_0) = 0$$ initial conditions (18)

$$\theta_1(x(T_3)) = m(T_2) - 1 \geq m(T_3) - 1 \geq 0$$ terminal condition of *m(t)* (19)

$$\theta_2(x(T_3)) = l(T_3) - 1 = 0$$ terminal condition of *l(t)* (20)

$$\theta_3(x(T_3)) = n(T_2) - 1 = n(T_3) - 1 = 0$$ terminal condition of *n(t)* (21)

$$\theta_4(x(T_3)) = \begin{cases} 0 & if\ T_2 \neq T_3 \\ Y(T_2) - A\int_{T_1}^{T_2}(1 - n(t))^{\iota}\, dt - A(T_1 + H - T_2) & if\ T_2 = T_3 \end{cases}$$ terminal condition of *Y(t)* (22)

where, T_0: office arrival time of the first commuter, T_1: the earliest work starting time, T_1+H: the earliest work ending time, T_2: the latest work starting time, T_2+H: the latest work ending time, T_3+H: the latest leave office time, $U(t_g) = c\, t_g - s(t_g)^{\eta}$: utility for commuting in the morning, $V(t_r) = - c\, t_r - r(t_r)^{\eta}$: utility for retuning home in the evening, $m(t)$: cumulative number of workers arrived office before time t, $l(t)$: cumulative office leave distribution - H shifted., $k(t)$: rate of start work. In these formula, $c =(e)$: slope of schedule cost, η: elasticity for congestion level, ι : elasticity of operation cost, ζ : cost parameter, A : technology level, and α: temporal agglomeration parameter are exogenous parameters.

The first term of right hand side of eq. (9) formulates aggregate utility about commuting in the morning which consists of schedule cost of departure time and congestion disutility,. The second term indicates the total fares just covering the operation cost in the morning. In the same way, the third and forth terms show aggregate utility and the total fares in the evening, respectively. The last term shows the total produced value from business activities that distributed for commuters as wage.

3.4 User equilibrium model under flexible time

If workers are told to choose office arrival time: t_g, office leave time: t_r, and work schedule: t, as they like without any political intervention, the distribution of such schedule would settle on an equilibrium situation, where no one can expect the improvement of his/her utility level by changing his/her schedule. In other words, the same utility level can be received regardless the timing. Let us focus on the i_{th} commuter and then the equilibrium condition can be formulated as follows, using $m^{-1}(i)$, $n^{-1}(i)$, $l^{-1}(i)$, which are inverse function of arrival time, work start time and leave office time distribution, respectively.

$$
\begin{aligned}
Y(n^{-1}(i)) \quad &+ \quad \left\{-s(m^{-1}(i))^{\eta} + c(m^{-1}(i))\right\} \\
&+ \quad \left\{-r(l^{-1}(i))^{\eta} - c(l^{-1}(i))\right\} - R \equiv W(i) = const.
\end{aligned} \tag{23}
$$

In order to derive the equilibrium schedule, we solve the optimal control problem that maximize social welfare as eq.(9) with the above equilibrium condition (23), as well as eqs.(10)--(22).

Because this problem maximize the same objective as the former S.O. problem over smaller feasible variable space, the solution of U.E.does not superior to the former S.O. problem. As discussed before, if railway fare level can be continuously differentiated by arrival/departure time, or office tax differentiated by work schedule can be introduced, the social optimum can be realized by responding behavior of commuters. Therefore, the difference of the social welfare level of the above two problems represents the economic effect of the idealistic policy, and the upper limit of realistic policies' economic effect.

4. Numerical example

4.1 Typical commuting patterns in system optimum problem

For the problems formulated in section3, we can get the analytical formula of the possible optimal solutions. Considering the possible combinations, there proved to be only three possible commuting patterns of optimal solution. However, they include integral constants which must be properly determined to satisfy boundary conditions of the differential equation system, and the genuine optimal solution can be derived after such parameters are set properly.

At first, we show the typical solution of the social optimum problem not as formula, but as numerical results. Figure 2 through 3 show three typical commuting and work scheduling patterns, which have possibility to be optimal solution. They share the same parameter values as follows besides the agglomeration parameter: α; $c=10.0$, $\iota=3.1$, $\eta=4.5$, $\zeta=0.0008$, $H=450$ and $A=44.4$. The value of A here certifies that $Y=20000$, when all workers begin to work at a point in time (conventional situation without flexible time).

(1) Total flexible pattern

When agglomeration effect is not strong ($\alpha = 0.2$), All workers utilize flexible time system; they start to work just after arrival in the morning and leave their office just after the end of the working time in the evening, as **figure 2**. There is no waiting loss and the schedule cost is minimized. Work stating time will distribute evenly in the 48 minutes interval between 8:36 and 9:24. This result owes to the weak agglomeration effect, which enables the spreading of work schedules effective for smaller congestions without severe loss of business efficiency. Flexible work pattern, that is, start work just after the arrival and leave office at end of work, can cut the schedule cost. In this case, average utility level is calculated as 19,184 (yen/day), which is the wage (19,816) minus congestion dis-utility (258) and railway fare (374).

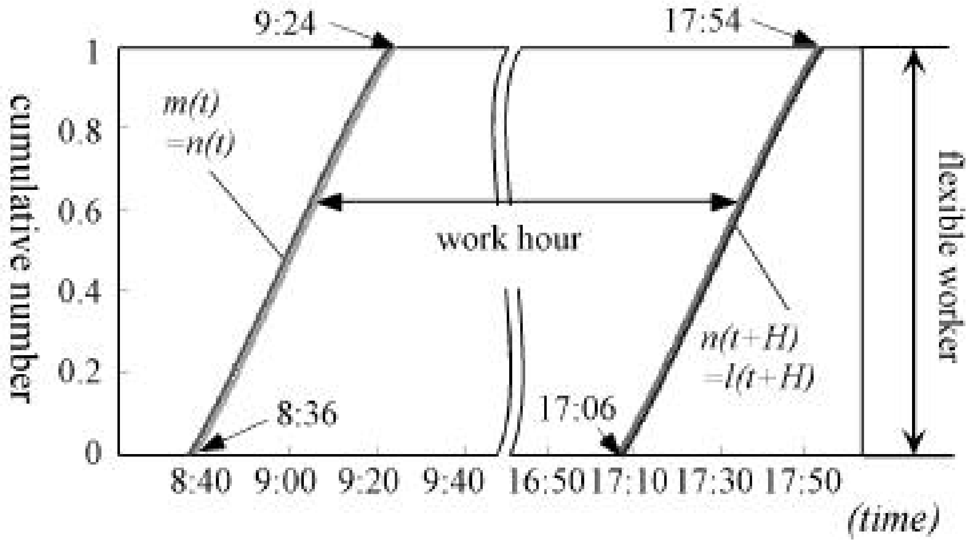

Figure 2　Total flexible commuting pattern

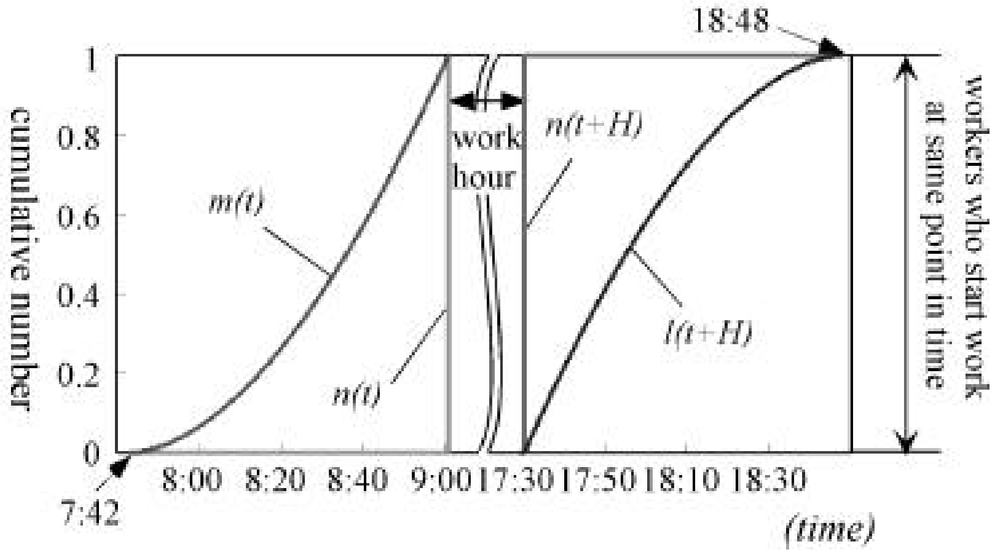

Figure 3　No flexible pattern

(2) No flexible pattern

Figure 3 shows the commuting pattern that maximize utility level under the condition that firms do not introduce flexible time. All workers are obliged to start to work at 9:00 in the morning and to work until 17:30 in the evening. In this setting, arrival time distribution and office leave time distribution are selected to maximize the utility level. The value of the wage and utility level are 20,000 and 19,000 (yen/day) respectively, regardless to the value of parameter α. As shown in figure 3, the first worker arrives at 7:42, 98 minutes before the work start time. Because arrival time distributes more flatly than the previous solution, both congestion dis-utility (182) and railway fare (264) become smaller than before. However, longer waiting time before and after working time

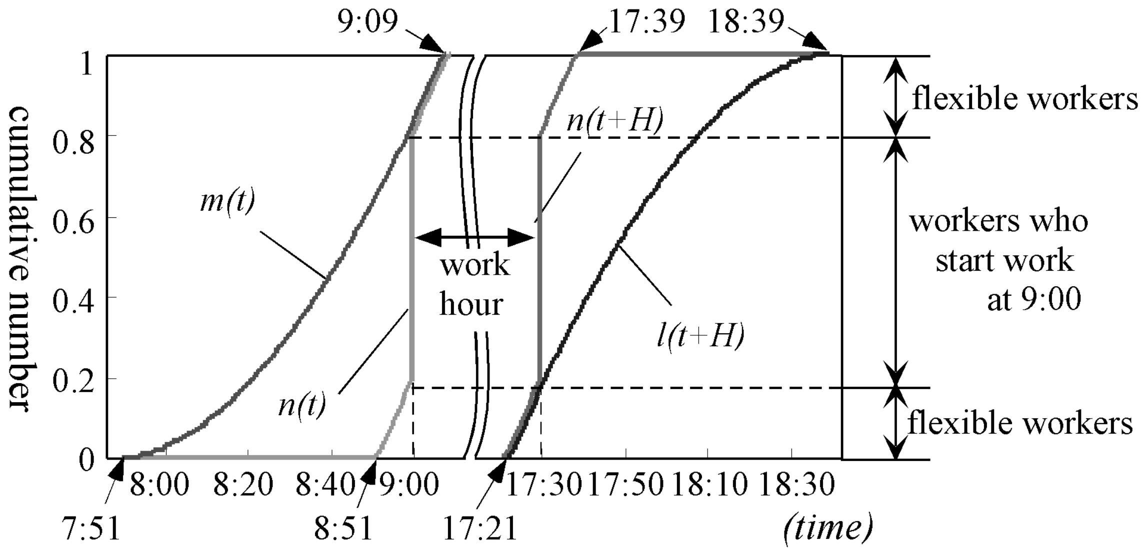

Figure 4 Mixture commuting pattern

yield schedule cost as much as 554 (yen/day), the utility level in result becomes 19,000 yen/day, 184 yen smaller than the total flexible pattern.

As shown afterwards in **figure 5**, this pattern without flexible work schedule cannot be chosen as the optimal pattern in any value of the parameter α, but when α is enough large, in other words, when work schedule coordination becomes important to enjoy high business efficiency, this pattern gives approximately same values as the optimal pattern.

(3) Mixture pattern

Figure 4 illustrates another possible pattern for optimal solution when agglomeration effect is not weak as the first case ($\alpha=0.5$, in this case). The solution is mixture of the above two patterns. Early in the morning between 7:51 and 9:00, people commute with low congestion level and arrive early. Because of temporal agglomeration, the commuters enjoy higher productivity and wage rate by synchronizing their work schedule with other people; then they wait the time to begin work until 9:00. Just after 9:00, lower congestion cost can compensate the loss of the productivity for delayed schedule. In this case, average utility level is calculated as 19,091(yen/day), still higher than the non flexible case (19,000). Division of the utility is as follows; wage: 19,917, schedule cost: 376, congestion cost: 183, and railway fare: 266. In this numerical setting, about 60% of the workers start to work at the same time. It means that the optimal pattern is realized by applying flexible time system to only 40% of the workers.

(4) Temporal agglomerations and optimal pattern

Figure 5 compares the utility levels of the above three possible patterns for several number of agglomeration parameter α. While the agglomeration effect is weak, total flexible pattern superiors to the other two patterns, because the flatter transport distribution reduces both congestion

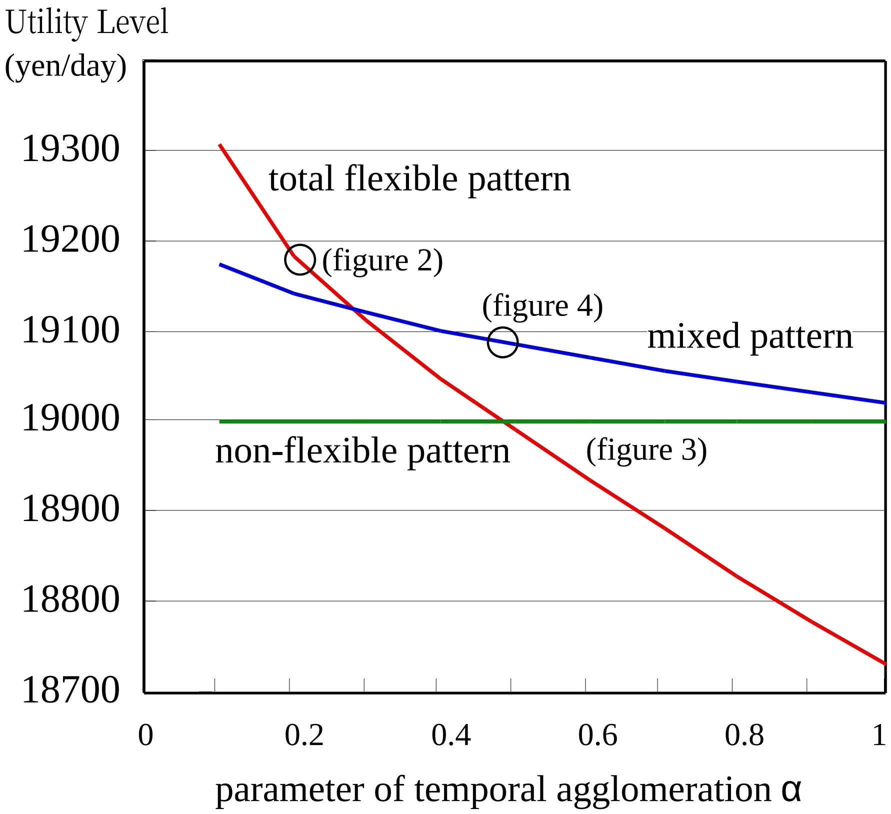

Figure 5 Utility levels of the three commuting patterns

cost and railway fare. When agglomeration effect becomes strong, the mixture pattern becomes optimum. As the parameter α becomes larger, this pattern keeps superiority to the non-flexible pattern, although the difference becomes smaller. This result can be understood by considering the following infinitesimal variation form the non-flexible solution; when all workers start work at 9:00, the train arriving just after 9:00 is totally vacant. If you use that train, you can reduce finite commuting dis-utility for infinitesimal decrease of wage rate and improve your utility. That means the non-flexible pattern is not optimal.

4.2 Commuting patterns in the user equilibrium problem

For the equilibrium problem, we can follow the same story just as system optimum problem and show that there are three possible commuting patterns, but because of the nonlinear feature of the equilibrium condition, eq.(23), we have not yet succeeded to get the numerical solution of mixture type. In the following part, we only show the comparison of (1) total flexible pattern and (2) non-flexible pattern. **Figure 6** compares the utility level of these two patterns, which show the similar characteristics with system optimum problem shown in figure 5. When agglomeration effect is small, total flexible pattern dominates, while agglomeration becomes stronger, flexible pattern gives ineffective result because of loss of productivity. Even in that case, we expect that there is a

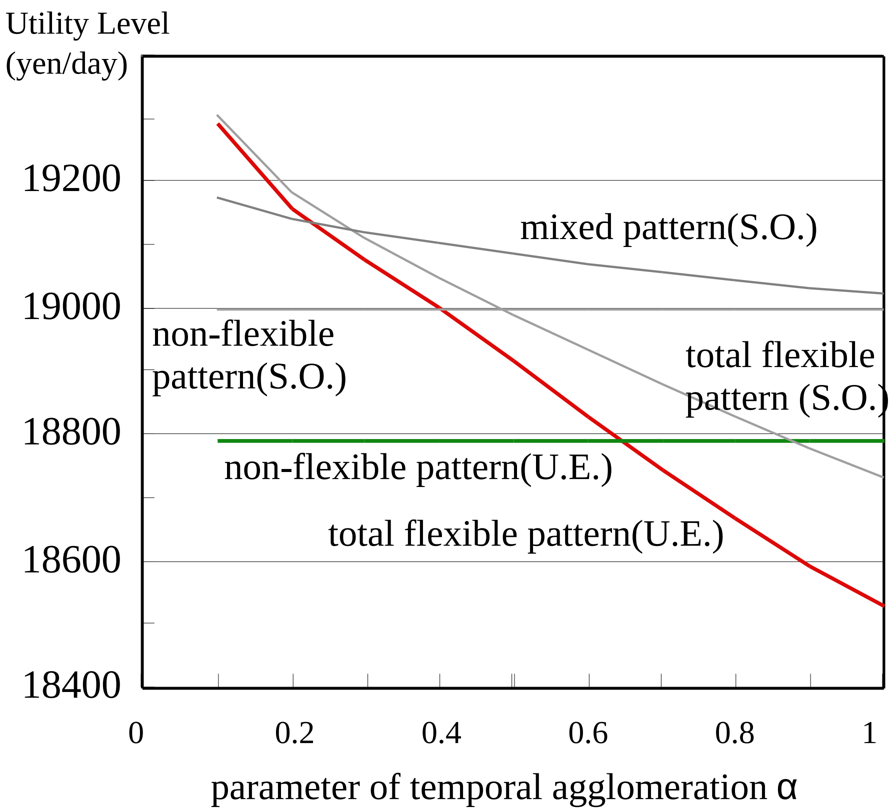

Figure 6 Utility level comparison among the commuting patterns

mixed pattern which is superior to the other two patterns, but we have not succeed to get the numerical result yet.

As discussed in 3.4, the system optimum problem affords the optimal solution better than the use equilibrium problem. In order to realize the system optimum solution, either time dependent railway fare system or office location tax differentiated by work schedule.

5. Concluding remarks

Peak leveling TDM policies have been hoped very much for long years in order to solve urban transportation congestion problems, but in real world, we seldom find the success of that policy. We have never investigated enough to answer the questions, such as; how large effect does such a policy have?, what conditions are required to get large effects from such a policy?

We have been trying to develop a methodology to analyze these questions. Fortunately, partial equilibrium models connected with optimal control theory can grasp the essential structure of the railway commuting service market, which is differentiated along departure time. Several studies have shown that these models are useful to investigate equilibrium commuting pattern, as well as social optimal pattern under TSM and TDM policies. Based on the difference of the objective functions, we can estimate the upper limit of such policies' effect. These models, therefore, provide a way to evaluate transport policies. Moreover, because most of solutions are derived analytically, we can investigate the effect of parameter changes on the effectiveness and applicability of each policy.

Needless to say, our models take very strong assumptions on player's behavior and exclude any personal differences. From viewpoint of travel behavior modelers, these models seem far from the reality. But owing to the simplifications, we get analytical tractability. If we combine more reliable parameter values by behavior analyses, we can prospect the market equilibrium and effect of policies more precisely. In this way, our approach is not substitute to travel behavior modeling, but rather complement.

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