A rank size rule, produced from a hierarchical branch office location model

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Abstract

There are a lot of studies on the rank size rule on cities, but few of them have succeeded to give a micro foundation on the rule. This study introduced a hierarchical branch office location model, and confirmed that rank size rule appears in a hierarchical structure of a company calculated by the model. As a result, it was shown that the power coefficient doesn't change through the decrease in number of branch offices, when fixed location cost becomes larger. On the other hand, it was shown that the power coefficient becomes larger and the employment becomes more evenly distributed, when the effectiveness of branch offices, in terms of the power to compress and aggregate business information, is increased. Due to the progress in information and communication technology, the effectiveness of branch offices would be enhanced, therefore, more flat organization of companies would be expected in the future.

Keywords: business service, industry location, hierarchical organization of company, rank size rule.

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1. INTRODUCTION

(1) Rank size rule and National planning

The rank size rule is an empirical proposition that the distribution of the population scale and the scale ranking of cities rides on a straight line when both plotted on logarithm axes. This empirical finding was firstly pointed out by Auerbach in 1913 and formulated by Zipf as old as 1949, statistical assessments are continued, even in recent years (such as Oveman and Ioannides, 2001).

Supposing that the scale ranking is y, supposing that the population scale of the city is x, a rank size rule is shown as follows.

$$\log y = \log A - \alpha \log x, \qquad (1)$$

where, α is called power coefficient and it shows the concentration of the city, while A is a constant to be regressed. The population accumulates to the small number of cities if the power coefficient α is small, while the population disperses in a lot of cities if the coefficient is large.

The monopolistic concentration of population and economical activities on the primal metropolitan area (frequently national capital), and concurrent out-migration from and decline of the remotely located rural area, became an important policy issue in many countries. If the rank size rule were strictly true, the feasibility of national development policies aiming to the development of rural cities might be problematic; because by the rank size rule, the number of cities possessing population over a certain threshold was already fixed. The development policy might become no different from the hopeless *competition game for the decreasing chairs*.

Hatta (2006) pointed out that the main cause of the monopolistic concentration to Tokyo around 1990 was the company's restructuring of central administrative function from Osaka, corresponding to the travel time reduction by the rapid train service of Shinkansen. We also agree that the main cause of nation-wide population distribution is company's business organization design. Especially the present development of information and communication technology (ICT) may be enhanced furthermore, and it may change the corporate organization. What kind of change is resulted in the rank size rule of the city system is an important issue in the national land policy.

(2) Existing researches about the cause of a rank size rule

A lot of empirical researches on the rank size rule have been accumulated for many years, but no stylized theory yet exist which successfully explains why the rank size rule is born.

Simon (1955) proposed a virtual dynamic process of city birth and growth: once a virtual new city of fixed population is born, that city is considered to be whether absorbed by an existing city with the probability proportional to the population of the existing city, or left alone as an independent city. Then Simon showed that the population distribution of the cities follows a rank size rule, when the probability of the birth rate of new city approached to zero. Krugman (1996) criticized about the assumption of the zero birth rate in this model. Duranton (2006) succeeded in giving a micro and economic foundation to the Simon's model but he based on strongly unrealistic assumption that all products of all cities have the same quality.

Gabaix(1999) showed that a rank size rule is concluded in the city system under the Gibrat's empirical rule of growth: the cities with similar scale have the similar proportional grown-up percentage. Ioannides and Oveman (2003) confirmed the Gibrat's rule in empirical data of the U.S. cities growths.

When seeing from the viewpoint of city systems, the rank size rule appears in the

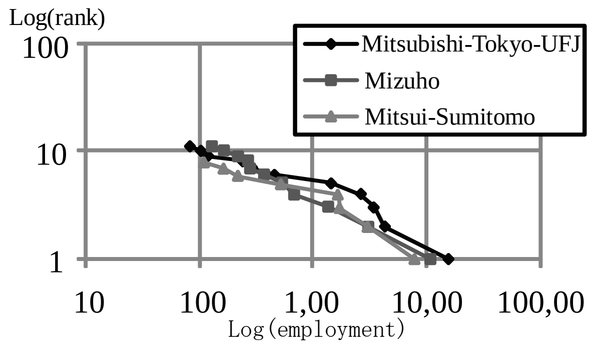

Figure 1 Rank size rule in area employment of Japanese mega banks

Christaller's city system. However, the central place theory including Christaller, is scarce about the economic foundations and is frequently criticized as "a mere geometry" only drawing a picture of a fact.

Fujita et.al. (1998) simulated an economic geography model and show the appearance of Christaller like city system structure, but he doesn't succeed in the reappearance of the rank size rule.

(3) Business organization of companies and the rank size rule

Pred (1976) insisted the importance of corporate organization as a cause of the structural changes in the city system. Recently, Mizuta (2008) aggregated the number of e-mail transmission logs in a company and analyzed them relating to the business organization of the company. He found that the e-mail transmission among business sections agrees with the hierarchical personnel organization structure with the president at top, and the number of e-mails from/to each section satisfies a rank size rule. If the transmission numbers of e-mail per each personnel are not so much different, the rank size rule observed in e-mail may be also in case for the in-house employment distribution in the company.

Figure 1 makes a graph of the area employment distributions of the top 3 companies of mega bank in Japan, based on the company's annual reports for investors, and it can confirm that those employment distributions approximately follow a rank size rule

(4) The purpose of this research

This research aims to build a model to describe the formation of a hierarchical structure in business organization, from a microscopic viewpoint. Cost minimization rule will be found to produce a rank size rule in the employment distribution. We do not assume any dynamic process in city level as Simon (1955). Instead, based on the static model in a company, a comparative static approach will analyze the plausible effects of the future ICT developments on the power coefficient of the rank size rule in the company's employment distribution.

2. HIERARCHICAL BRANCH OFFICE LOCATION MODEL

(1) Existing research of the hierarchical branch office location model

Some studies (Hino, 1999, Suda, 1998) have tried to model the branch office location of a company and to compare the calculated results with the real distribution of the branch office. The authors of this paper have also engaged on such branch office location model: applying a model to assess the regional effects of inter-city transportation development projects (Tsukai, Okumura, 2003), and expanding the model by considering the uncertain fluctuation of jobs (Okumura, Tsukai, 2008). However, all these models only consider a problem whether branch offices at one middle layer should be inserted or not, between the customers and the headquarter office. They cannot describe a multi-layer branch office structure permitting that an upper-layer branch office manages several substratum branches.

On the other hand, Sahin (2007) classified the researches of the multi-layer facility location model in the electric communication network research, and showed that the number of hierarchies is predetermined in all those models. A logistic center location model by Kijimanawat and Ieda (2004) is an exceptional study, which endogenously determines the number of hierarchies, but their model cannot describe the tree structure rooted by a headquarters.

(2) Business organization structure of a representative company

Irrelevant to the previous branch office location models, this study propose a model, which endogenously decides both the number of hierarchies and the locations of branch offices in an organization network rooted by a headquarter.

We consider a representative company in business service industry. Hereto after, we explain the assumptions of the model.

1) This company provides business service to the all customers spatially distributed over the whole country. This company is composed by one headquarter office and several number of branch offices on the different layers.
2) Branch offices on the first (bottom) layer only control the direct communications with customers. Branch offices (and the headquarter office) on the upper layers, manage the linked substratum branch offices, and possibly, communicate directly with the customers nearly around the office.
3) They exchange business information through face-to-face contacts between the front office and the customers, between the linked offices. Such information exchange requires certain transportation cost reflecting the distance between the locations.
4) A branch office takes a role to compress and aggregate the business information gathered from the substratum branch offices or from customers. In other words, the branch office screens the numerous informations from the lower layers, and only pass through to the upper layer, difficult business problems included in the gathered information with a fixed proportion. We define such percentage of information to be sent the upper layer as “information concentration ratio”, indicated by R.
5) Corresponding to the business information quantity which each branch and headquarter office will handle, employment and location cost are required accordingly.

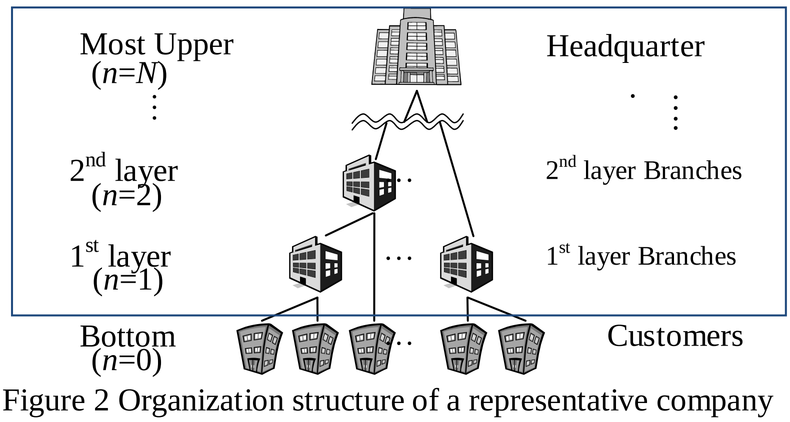

Figure 2 Organization structure of a representative company

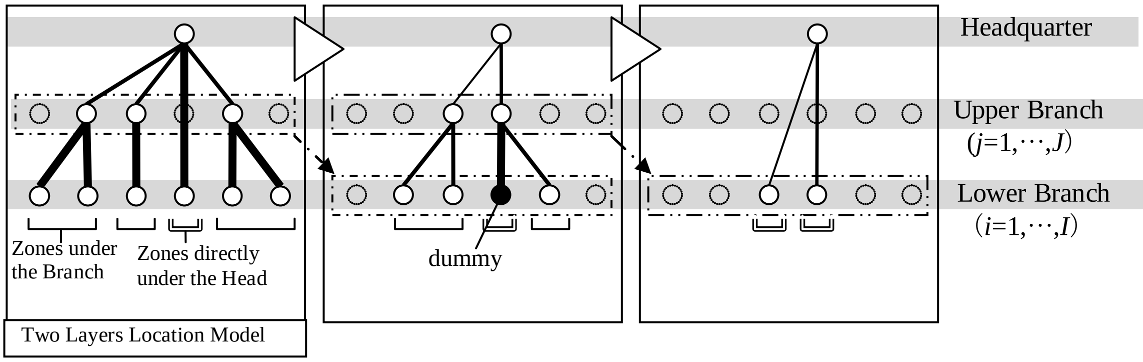

Figure 3 Calculation procedure of multi-layer hierarchies by the two-layers location model

The tree structure of the company including one headquarter office, branch offices, is shown in **Figure 2**.

(3) The computation procedure

The multi-layer hierarchical branch office location model is based on the two layers branch location model by Okumura and Tsukai (2008) and repeatedly use that model from the most bottom layer, up to the headquarter. The two layers branch location model judges whether you locate an upper-layer branch office in order to economize the communications between the substratum branch office and the headquarter office or not. The computation procedure is explained in **Figure 3**.

1) Initially, as the first iteration, two layers location model is applied with considering each customer as a substratum branch office, in order to judge the need of the first layer branch offices.
2) In the nth iteration, we multiply R with the information from the branch offices located at the previous $n-1$th iteration, and consider them as the substratum branch offices at the nth iteration. On the other hand, for the branch offices (or customer) already managed by a direct link to the headquarter in the previous $n-1$th iteration, place a dummy branch substratum office. Following these preparations, the two layers location model is applied.
3) Such procedure will be iterated. Through the iterations, the cost reduction effect of a new branch office becomes smaller. At last, any additional location of a new branch cannot cover the location cost. The iteration process stops there.

(4) Formulation of the two layers branch office location model

This two layers location model is formulated to determine the additional inserting of upper branch offices resulting to the total cost reduction through the compression of business information exchanges between the headquarter and the substratum branch offices located in the $n-1$th iteration.

1) Location cost of the headquarter office

When the employment size of the headquarter office is large, wider space must be prepared. Then, the location cost of the headquarter office: C_0^n is given as follows;

$$C_0^n = f_0 + (h_0 + p_0)s_0^n, \qquad (2)$$

where, f_0: fixed location cost of the headquarter office, h_0: wage per one employment in the headquarter, p_0: floor space rent per one employment, and s_0^n : employment at the headquarter in the nth iteration.

2) Location cost of branch offices

Similarly, the location cost for the upper layer branch offices, located in the n th iteration C_1^n is given as follows;

$$C_1^n = \sum_{j=1}^{J} \{ fX_j^n + (h_j + p_j)s_j^n \}, \qquad (3)$$

where, f : fixed location cost of a branch office, h_j : wage per one employment at the branch office in city j, p_j : floor space rent in city j per one employment, and s_j^n : employment at the branch office in city j.

3) Controllable variables

X_j^n is a 0-1 controllable variable indicating the location of the upper layer branch office in city j.

$$X_j^n \in \{0,1\} \quad \forall j. \qquad (4)$$

The number of employment at branch office s_j^n is such determined that meets the needs from the linked substrata branches i, then satisfies the following;

$$s_j^n = \sum_{i=1}^{I} \lambda_i^n Y_{ij}^n \quad \forall i, j, \qquad (5)$$

where, λ_i^n : total business information quantity from substratum branch office in city i. When we introduce a control variable Y_{ij}^n, indicating the quota of the information heading for upper branch j, from the lower branch office i. It is limited by the existence of upper branch office X_j^n in city j as follows;

$$0 \le Y_{ij}^n \le X_j^n \quad \forall i, j. \qquad (6)$$

Accordingly, when $X_j^n = 0$, both s_j^n and C_1^n must be zero.

4) Information exchange cost

Next, let us consider the information exchange cost between the linked upper and lower branch offices. We define the unit of business, based on the volume requiring one unit of information exchange, then total information exchange cost C_2^n can be given as follows;

$$C_2^n = \sum_{j=1}^{J} \sum_{i=1}^{I} \lambda_i^n d_{ij} Y_{ij}^n, \qquad (7)$$

where, d_{ij} : information exchange cost between upper and lower branch offices for one employment, given exogenously according to the transportation conditions.

The business information which was collected at the upper branch office is concentrated with rate of R, then exchanged further with the headquarter office. Total information exchange cost between the upper branches and the headquarter is given as follows;

$$C_3^n = R \sum_{j=1}^{J} s_j^n d_{j0}, \qquad (8)$$

where, R : information concentrate ratio, and d_{j0} : information exchange cost between upper branch and headquarter offices for one employment, given exogenously according to the transportation conditions

If the lower branch office location is not far from the headquarter location, direct information exchanges between the lower branch and the headquarter becomes inexpensive than locating the additional upper branch office in middle. In that case, all of the business exchange from lower branch i occurs directly to the headquarter office.

Total cost of such direct exchanges C_4^n is given as follows;

$$C_4^n = \sum_{i=1}^{I} \lambda_i^n d_{i0} Z_{i0}^n , \tag{9}$$

where, d_{i0} : direct information exchange cost between a lower branch and the headquarter office for one employment of the lower branch office, Z_{i0}^n: a controllable variable showing the share of headquarter exchange from the lower branch office in city i.

$$0 \le Z_{i0}^n \le 1 \quad \forall i \tag{10}$$

5) Cost minimization problem

The company is considered to minimize the total cost C_T^n, composed of the cost components stated above, and the problem is formulated as follows;

$$\min_{X_j, Y_{ij}, Z_{i0}, s_j, s_0} \quad C_T^n = C_0^n + C_1^n + C_2^n + C_3^n + C_4^n \tag{11}$$

$$s.t. \quad X_j^n \in \{0,1\} \quad \forall j \tag{12}$$

$$\sum_{j=1}^{J} (Y_{ij}^n + Z_{i0}^n) \ge 1 \quad \forall i \tag{13}$$

$$0 \le Y_{ij}^n \le X_j^n \quad \forall i, j \tag{14}$$

$$0 \le Z_{i0}^n \le 1 \quad \forall i \tag{15}$$

$$s_j^n = \sum_{i=1}^{I} \lambda_i^n Y_{ij}^n \quad \forall i, j \tag{16}$$

$$s_0^n = \sum_{i=1}^{I} \lambda_i^n Z_{i0}^n + R_C \sum_{j=1}^{J} s_j^n \quad \forall i, j \tag{17}$$

Where, (13) shows the constraint condition that in each lower branch office, intractably difficult works are fully covered under the control either of an upper branch office or the headquarter office. Eq. (17) shows that the total employment at the headquarter must satisfy the demand both from upper branch offices and lower branch offices.

(5) The result employment distribution

After the iterated computations, the total number of employment at branch office and the headquarter are given respectively as follows;

$$S_j = \sum_{n=1}^{N} s_j^n \quad \forall j , \tag{18}$$

$$S_0 = s_0^N , \tag{19}$$

where S_j : total number of employment at branch office in city j, S_0: total number of employment at the headquarter office.

3. ANALYSYS OF JAPANESE COMPANY’S ORGANIZATION

(1) Setting of a representative company and parameters

We consider a representative company having demand of service equivalent for 1,000 employment at the bottom front facing to the customers, from all over the country of Japan. Spatial distribution of the customers is proportionally set to the general size of economic activities, which is given by the total number of employment for all industries in each of 194 zones in year of 2004, as shown as **Figure 4**. Location of the headquarter office is fixed at downtown Tokyo (Tokyo 23 Wards area).

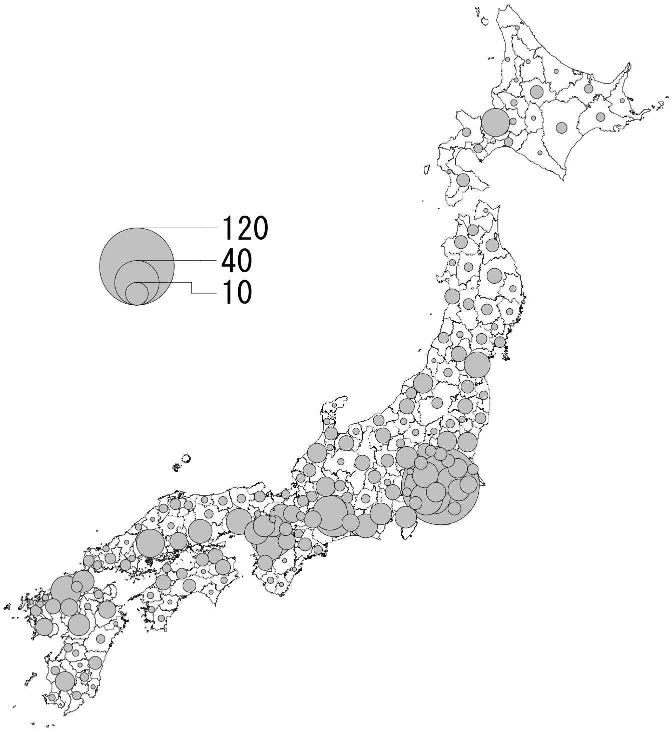

Figure 4 Demand distribution in 194 zones

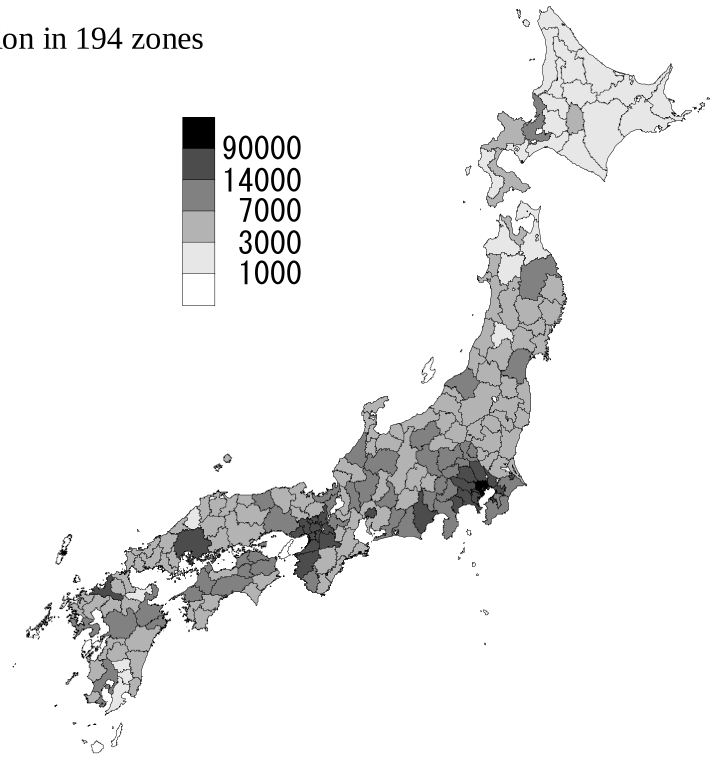

Figure 5 Floor rent for one office worker in 194 Zones

Each branch office worker uses 15.2m^2 of floor space, irrespective to the geographical location. Land rent is set by the empirical land-rent data at the city, which has largest amount of employment within each zone, as shown in **Figure 5**. With contrast to the land rent, we neglect the spatial difference of worker's wage rates, and set the value as 2,960,000 yen/year, irrespective to location. Inter-zonal transportation cost was given from the comparison of generalized cost including time of the railway shortest time route and that of the rail-air shortest time route. Time value of one hour is considered as 3,000

Table 1 Number of the located offices

Fixed location cost	Information Concentrate Ratio R										
f (10000yen)	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
2,000	44	41	37	35	30	25	17	7	2	1	1
3,000	34	31	28	22	19	14	11	5	1	1	1
4,000	25	22	19	16	15	13	9	4	1	1	1
5,000	20	16	15	13	13	11	8	4	1	1	1
6,000	16	13	13	12	12	10	5	4	1	1	1
7,000	13	13	10	10	9	9	4	3	1	1	1
8,000	11	10	9	9	9	7	4	2	1	1	1

yen.

(2) Computation result of the number of branch offices and organization structures

The reminders of the exogenous parameters are the information concentrate ratio R, fixed location cost of headquarter office f_0, and fixed location cost of branch office f. In order to make the analysis simpler with two parameters, we assume $f = f_0$. Because we prefix the location of the headquarter in Tokyo, parameter f actually gives any effect to the result.

For several combinations of those two parameters of (R, f), the multi-layer hierarchical location model is applied to find the number and locations of branch offices. **Table 1** shows the number of the offices located in Japan including both the headquarter and branches (if any). Either increase of fixed location cost f or increase of the information concentrate ratio R gives the smaller number of branch offices. The same number of branch offices is resulted in several different combinations of the two parameters, but the actual geography of the organization structure is different. For example, we show the organization structures containing 11 offices, obtained from $(f, R) = (3000, 0.6)$ and $(f, R) = (5000, 0.5)$ in **Figures 6** and **7**, respectively. In those figures, 194 zones are arranged in the horizontal axis in North-South order, and the total employment at each office is plotted on the vertical axis. Superscript number on each branch location shows the hierarchy, obtained as the largest iteration number in the calculation when that location keeps a branch office. Straight lines in the figure mean the management linkages.

When we compare the Figures, case of smaller fixed location cost and larger information concentrate ratio (**Figure 6**) fosters larger employment at the Tokyo headquarter office but fewer employment in the branch office in Osaka. On the other

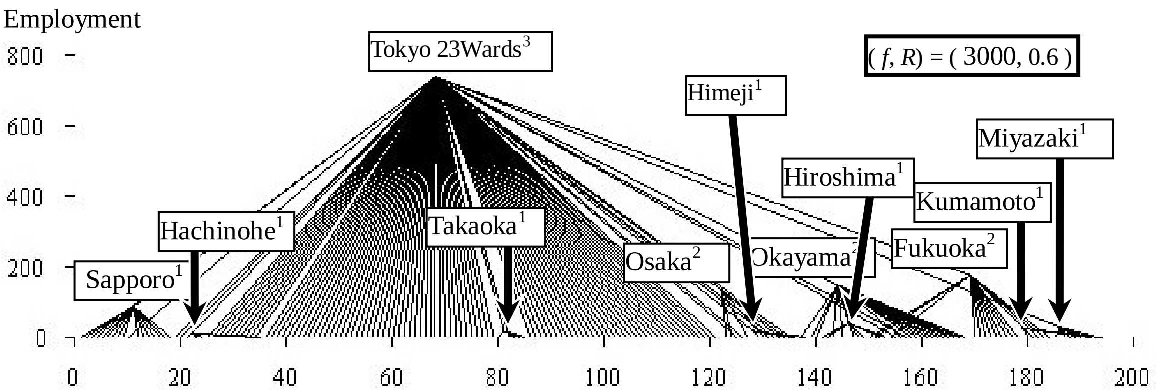

Figure 6 Organization structure (low fixed cost and mild concentration)

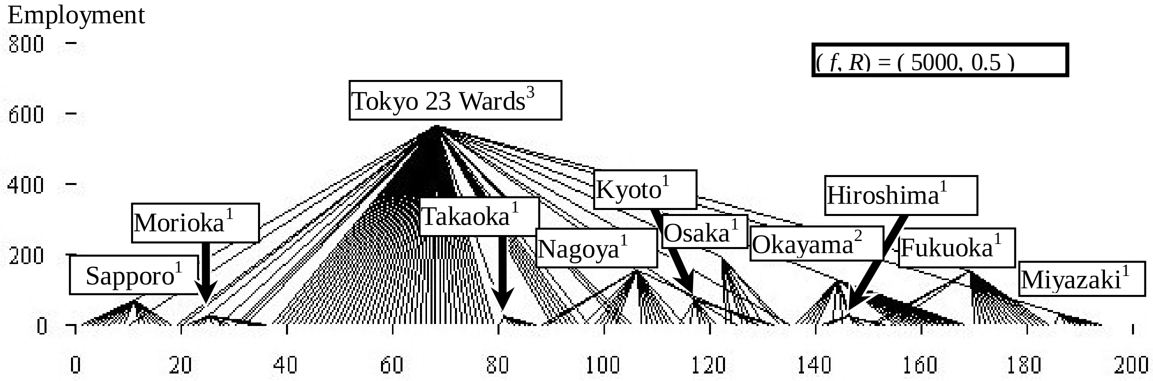

Figure 7 Organization Structure
(high fixed cost and strong concentration function of branch offices)

hand, Himeji, Hiroshima and Kumamoto are arranged respectively in the substratum under Osaka, Okayama and Fukuoka, where the hierarchy number is 2. In **Figure 7** in case of expensive fixed location cost and more efficient branch office technology, branches in Himeji and Kumamoto disappear and hierarchy number of Osaka and Fukuoka become one. The resulted organization structure seems flatter.

4. THE RANK SIZE RULE FOR THE EMPLOYMENT

(1) The comparison of the inclinations

Concerning on the several combinations of (f, R) having 11 locations of branch and headquarter offices, we draw the rank size relationships both on logarithm axes, as **Figure 8**. In this figure, we found larger value of the power coefficient in the rank size rule, showing the steeper inclination of the graph, when fixed cost is larger and information concentration is stronger.

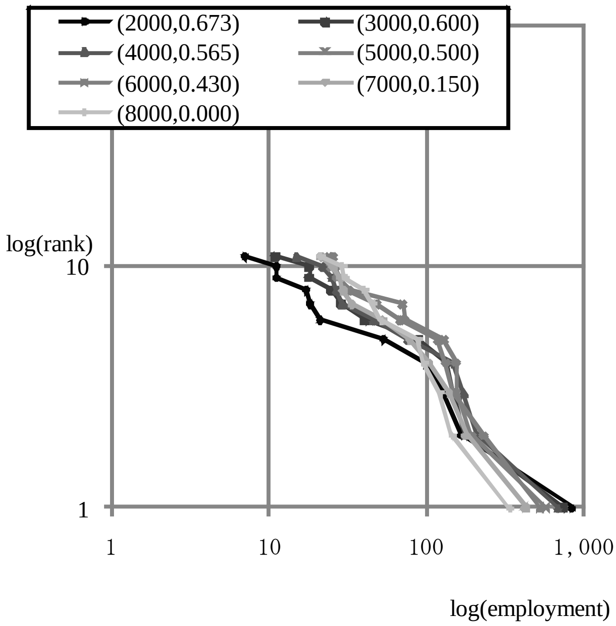

Figure 8 Rank-size of employment for cases of 11 offices

(2)The sensitivity analysis for the information concentrate ratio

We keep the fixed location cost parameter equal 20,000,000 yen, then we draw rank size relationships in employment for several values of the information concentrate ratio R, as **Figure 9**. When concentrate ratio is small, it means concentration function of branch office is strong, the power coefficient becomes large and the inclination of the graph becomes steep. This is because smaller amount of information is sent upwards, the required employment at upper branches and the headquarter becomes smaller.

(3)The sensitivity analysis for the fixed location cost

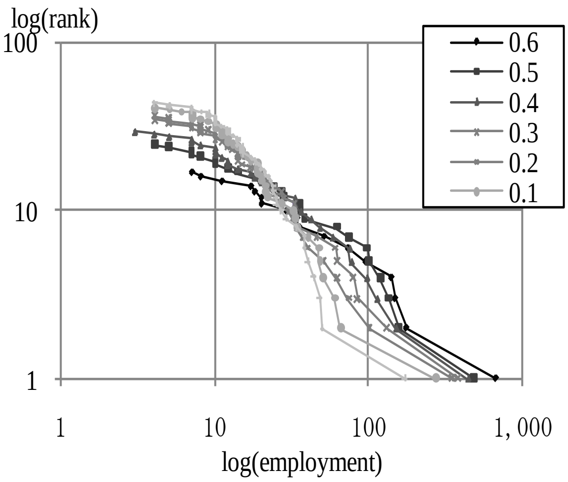

Figure 9 Rank-size of employment for various *R*

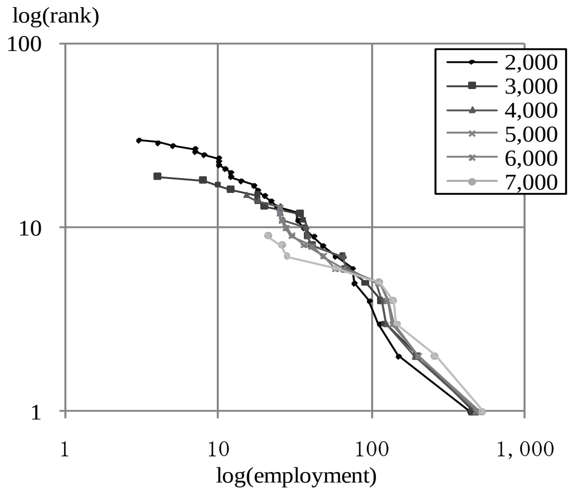

Figure 10 Rank-size of employment for various *f*

On the other hand, we change the value of the fixed location cost f, while keeping the information concentration ratio constant as 0.4. **Figure 10** shows the rank size relationships in employment for several values of the fixed cost parameter. As a result, no changes are observed in the inclination of the graphs, while number of branch offices may differ.

(4)The change of the corporate organization

Let us check the changes in the corporate organization structures due to the changes of parameter value. **Figure 11** shows the case of $(f, R) = (3000, 0.5)$, which is different in fixed location cost f from the case of **Figure 7**. Branch offices of hierarchy one newly

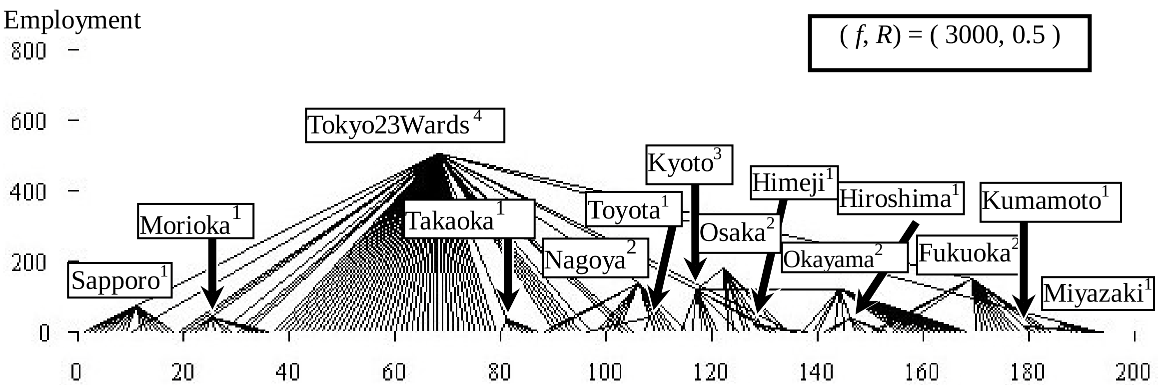

Figure 11 Organization Structure *(f,R)=(3000,0.5)*

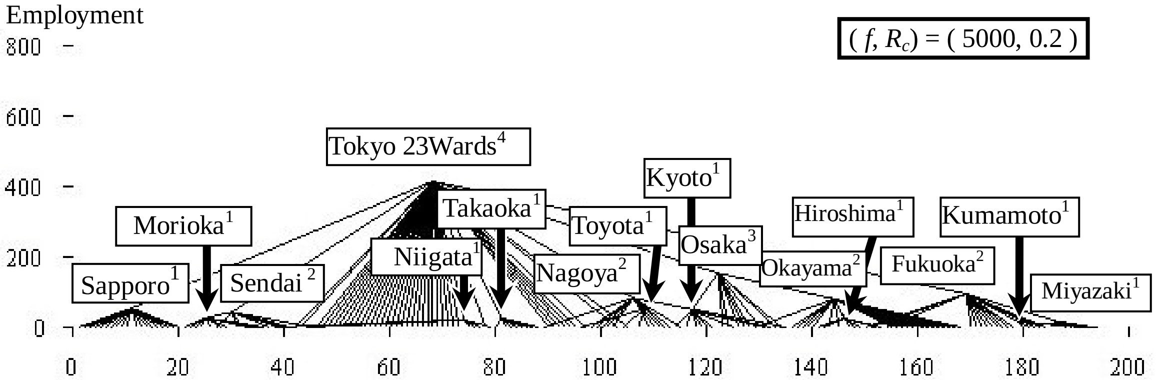

Figure 12 Organization Structure *(f,R)=(5000,0.2)*

appeared under Nagoya, Osaka and Fukuoka, but a new branch office does not appear in the upper layer. On the other hand, **Figure 12** shows the case of $(f, R) = (5000, 0.2)$, which differs in R from the case shown in **Figure 7**. In this case, upper branch office at Sendai appears over Morioka branch. On the same time, new branch office is inserted at Niigata, in order to manage 38, 44, 74-77, 79 zones, which had been directly managed by the headquarter office in Tokyo. In that way, the change of the power coefficient in rank size rule corresponds to organization changes other than the first bottom layer.

5. CONCLUSION

This study proposed a multi-layer hierarchical branch office location model and showed that the rank size rule appears in the employment in the hierarchical organization of a company. Sensitivity analysis was done for the parameters of fixed location cost and information concentrate ratio.

As a result, if the concentration function becomes stronger in branch office due to development of ICT (smaller R value) in the future, the power coefficient becomes larger, then result in more flatter business organization appears. However, there is the different scenario of ICT development; it results in the qualitative change of the exchanged information, the branch office relies more frequently on the upper organization. That may cause of increase of R value, that is the opposite exogenous change from our analysis above.

Organizational changes analyzed here may further alter the demand of inter-city transportation pattern and service. Such effects will expand further to the macro-scopic changes of system of the cities. We must continue to analyze such causations and try to give a theoretical basement for the recent observations that the power coefficient seems larger in developed countries, such as Soo (2005) and Rossi and Wright (2007).

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