ANALYSIS OF RELATIONSHIP AMONG DEMAND DISTRIBUTIONS AND URGENCY OF A PERISHABLE ITEM IN RELIEF DISTRIBUTION

Rubel DAS[1] and Makoto OKUMURA[2]

[1]Member of JSCE, Assistant Professor, International Research Institute of Disaster Science, Tohoku University
(468-1 Aoba, Aramaki, Aoba-ku, Sendai 980-0845, Japan)
E-mail:rubeldas@irides.tohoku.ac.jp
[2]Member of JSCE, Professor, International Research Institute of Disaster Science, Tohoku University
(468-1 Aoba, Aramaki, Aoba-ku, Sendai 980-0845, Japan)
E-mail:mokmr@m.tohoku.co.jp

Urgency and perishability are highlighted during relief operation after the 2011 great east Japan earthquake. Perish rate becomes higher because of several reasons including poor preserving facilities. This paper presents a dynamic programming model to optimize decisions (i.e., "how much and when to order") for replenishing a perishable item facing declining demand and urgency. An exponential distribution of declining demand is adopted. The results of the model for exponential distribution demand is compared with that for linear declining demand. The proposed model exhibits a variation in replenishment intervals and order quantity. Herewith, total cost of inventory changes for urgency and perishability. In addition, the trend of delayed satisfied demand in planning horizon have patterns that do not depend on parameter values rather depends on declining demand distribution type.

Key Words : *urgency, perishability, demand distribution,, disaster logistics*

1. INTRODUCTION

Mathematical modeling for disaster logistics (DL) was introduced couple of decades ago by Knott (1988). Altay and Green (2006) and Galindo and Batta (2013) provided a holistic review of the Operational Research/Management Science (OR/MS) models for DL until recent years. Both resources iterated the importance of mathematical models for DL. We have identified two issues that has not gained attention properly. First, relief demand declines over time. DL mathematical models consider a constant or uniform distributed demand throughout the relief operation period. However, the demand for relief items declines and the relief operation terminates when demand becomes too little (theoretically when the demand becomes zero).

Another important aspect of post-disaster scenarios is "urgency," which represents the degree of effectiveness of a relief item at the time of delivery. It is natural to think that higher urgency product should be delivered earlier. Moreover, the urgency for a particular relief item is not constant over time. This phenomenon is observed after 2011 GEJE. There was high demand for blanket aftermath of the 2011 GEJA. However after couple of weeks the weather warmed up and the urgency of blanket goes down. But the donors sent a large amount of blanket to affected areas. The authorities in affected areas faces difficulties in processing of the leftover blankets. Sheu (2007 and 2010) introduced a novel approach of relief allocation depending on relief urgency.

These two unrecognized issues are applied for a perishable item. A perishable item is subject to a continuous loss in their masses or utilities throughout their lifetime due to decay, damage, spoilage, and plenty of other reasons. In practice, perishability is relevant to several relief items, such as fruits, medicine, blankets, and others. Some relief items (i.e., fruits) perish because of their characteristics and others perish because of improper warehouse facilities.

Beamon and Kotleba (2006) developed an operational model of inventory ordering strategies in which demand is characterized as uniformly distributed. Lodree and Taskin (2008) formulated the inventory planning problem encountered by donor organizations using variants of the news-vendor model. Proactive actions to maintain inventory levels are compared with financial investment in an insurance policy. Demand is described as having a uniform distribution in the model. Das and Hanaoka (2014) extended the relief inventory model to consider stochastic demand and lead time for a large-scale disaster. Above mentioned two issues are interconnected and influence inventory management. One of the major concerns of inventory management is deciding on when and how much to order to minimize the total cost. This paper aims to develop a dynamic programming model for a perishable relief item using the following assumptions: a constant perish rate, declining demand, allowing backlogging with penalty, and considering the urgency. The solutions to the model determine an optimal replenishing schedule during a finite planning horizon to ensure minimum total cost associated with the inventory system. The proposed model exhibits variations in both the replenishment cycle length and the ordered quantities.

The remainder of the paper is structured as follows. Section 2 consists of several sub-sections that explain the proposed model and a solution algorithm. Section 3 presents a numerical analysis using the proposed model and reports the results. Section 4 presents the conclusion of the paper and the summary of the study outcomes.

2. METHODOLOGY

Consider a large-scale earthquake has damaged a vast area. Relief items available in the affected areas are either destroyed by the earthquake or quickly depleted, necessitating the rapid deployment of relief items to reduce further human suffering. A manager plans for distributing a relief item in damaged area through a planning horizon, *H*. In the proposed system, no inventory is held at the beginning and at the end of replenishment cycle *[j,k]* along the planning horizon. The cycle starts with accumulating shortages from time *j* until time *p*, at which point a replenishment is scheduled. In this study, *[j,p]* is called the out of stock duration and the quantity of relief item during this period is called delayed satisfied demand. The quantity of the relief item received at time *p* equals the sum of the demand backordered during period *[j,p]* and the demand requirement in period *[p,k]*. The relief item is considered with a constant perish rate (θ) over time. Perishability of units occurs only when the item is effectively in stock, and no replacement of perished units occurs.

The objectives of the proposed model are to identify the locations of *p* along the planning horizon and order quantity at each *p*. In other words, it aims to determine the optimal sequence of replenishment point *p* that minimizes total costs. The total cost consists of holding, shortage, operational, and setup costs. Per unit costs for holding, shortage and operational cost are H_0, P_0 and C_0 respectively. The setup cost for each order is A_0. Inventory is continuously reviewed and replenishment is instantaneous; i.e., replenishment capacity is infinite and lead time is zero. Relief item shortage is completely backordered and incurs a shortage cost (or penalty cost). The realized shortage cost depends on urgency that has two parameters gamma (γ) and mu (μ).

The demand decreases exponentially during the planning horizon of length H:

$$D(t) = a_0 e^{-a_1 t} \text{ when } 0 \le t \le H \tag{1}$$

where a_0 and a_1 are constant.

(1) Holding cost

Let us consider the inventory level at time *t*, *I(t)*, during a cycle, that is, $j \le t \le k$. This inventory level declines gradually by the combined effect of demand and the perish rate. Note that no perishability effect occurs if no inventory is on stock. Therefore, the variation of *I(t)* with respect to time is:

$$\frac{dI(t)}{dt} = -I(t)\theta t - D(t)\,,\; j \le t \le k \tag{2}$$

By multiplying $e^{\theta t}$ on both sides of eq (2), integrating by parts and replacing the demand function by eq (1) results in (Benkherouf, 1995):

$$\text{Inv. Quantity, } I(t) = e^{-\theta t}\int_t^k a_0 e^{-a_1 t} e^{\theta t}\, dt \tag{3}$$

Now, H_0 represents the holding cost per unit time. Therefore, the holding cost (*HC*) for a cycle is written as:

$$HC = H_0\, p\int_p^k e^{-\theta t}\int_t^k a_0 e^{-a_1 t} e^{\theta t} dt dt \tag{4}$$

$$= \frac{H_0 a_0}{\theta - a_1}\int_p^k (e^{(\theta - a_1)k} e^{-\theta t} - e^{-a_1 t})dt \tag{5}$$

Eq (5) becomes easy to integrate.

(2) Shortage cost

If relief items cannot be delivered to victims on time, the system incurs a shortage cost. This cost is larger when urgency value is high. We introduce “realized shortage cost” by incorporating urgency, and this realized shortage cost changes with urgency. The realized shortage cost of a particular relief item is higher when that item has a higher urgency. If the shortage cost per unit item is P_0 and shortage quantity is S, the realized shortage cost is equal to:

$$RS(P_0, S) = (i + \gamma e^{-\mu t}) P_0 S \tag{6}$$

Where $RS(P_0, S)$ represents the realized shortage costs incurred and $1+\gamma e^{-\mu t}$ represents the urgency with parameters γ and μ. It shows that urgency declines gradually, which indicates that urgency starts declining in the aftermath of a hazard. Therefore, urgency in the temporal dimension is required for designing inventory planning.

The shortage quantity is satisfied after arrival of relief at the next replenishment point. The shortage quantity can be imagined as delayed satisfied demand, and is given at time t by:

$$\text{Quantity, } S(t) = \int_j^t a_0 e^{-a_1 t}\,dt\,,\ j \le t \le p \tag{7}$$

Now, P_0 is the shortage cost per unit attributable to relief item shortage. Because we assume that urgency decreases exponentially in Eq (7), realized shortage cost also changes accordingly. The highest realized shortage cost is incurred during the aftermath of a disaster.

Realized shortage cost,

$$RS = P_0 \int_j^p (1 + \gamma e^{-\mu t}) \int_j^t S(t)\,dt\,dt \tag{8}$$

$$= P_0 \int_j^p (1 + \gamma e^{-\mu t}) \int_j^t a_0 e^{-a_1 t}\,dt\,dt \tag{9}$$

Eq (9) is easy to integrate.

(3) Operational cost

A cycle starts with accumulating shortages from time j to time p, at which time replenishment is scheduled. The total amount of relief items ordered at time p equals the sum of the demand backordered during period $[j,p]$ and the demand requirement in period $[p,k]$.

$$\text{Ordered quantity, } Q(P) = I(P) + S(P) \tag{10}$$

Now, C_0 is the operational cost per unit.

$$\text{Operational cost, } OC = C_0 Q(P) \tag{11}$$

After replacement with eq (7) and eq (3)

$$OC = C_0 (e^{-\theta p} \int_p^k e^{\theta t} a_0 e^{-a_1 t}\,dt + \int_j^p a_0 e^{-a_1 t}\,dt) \tag{12}$$

Eq (12) becomes easy to integrate.

(4) Total cycle cost

Total cycle cost (W) consists of realized shortage cost (RS), holding cost (HC), operating cost (OC), and setup cost (A_0):

$$W(j, p, k) = A_0 + HC + RS + OC \tag{13}$$

Now, replacement with eqs (5), (9), and (12) for further computation.

For given j and k, the optimal replenishment point p in the cycle is differentiated. Because the value of p depends on the value of j, we replace $k=j+\alpha$ and $p=j+\beta$, where $\alpha>\beta$. After replacing the k and p values, we differentiate with respect to β and the outcome set equal to zero for locating β:

$$\frac{\partial W(j,p,k)}{\partial \beta} = 0 \tag{14}$$

After several algebra operations on eq (14):

$$\begin{aligned} W_\beta(j,\beta,k) = {} & \frac{H_0 a_0}{\theta - a_1}(-e^{(\theta - a_1)(j+a) - \theta(j+b)} + e^{-a_1(j+b)}) + \\ & P_0(-\frac{a_0}{a_1} e^{-a_1 j - a_1 b} + \frac{a_0}{a_1} e^{-a_1 j} - \frac{a_0 \gamma}{a_1} e^{-a_1 j - \mu j} e^{(-a_1 - \mu) b} \\ & + \frac{a_0 \gamma}{a_1} e^{-a_1 j - \mu j - \mu b}) + C_0(-\theta \frac{e^{-\theta(j+b)} a_0}{\theta - a_1} e^{(\theta - a_1)(j+a)} \\ & + a_1 \frac{e^{-a_1 j - a_1 b} a_0}{\theta - a_1} + a_0 (e^{-a_1(j+b)})) \end{aligned} \tag{15}$$

Because obtaining a closed-form solution to eq (15) is difficult, the replenishment point p ($p=j+\beta$) is estimated using a numerical search technique. A bisection algorithm is used to find the location of β^* (hence, p^* for given j).

(5) Sequences of replenishment points

The objective of the proposed model is to determine the optimal sequence of replenishment point p along the planning horizon H that minimizes total system costs. The optimal sequence of replenishment

Table 1 Description of scenarios and results

Scenario description			Results				
	γ	θ	Total cost	Out of stock	Shortage cost	Holding cost	per-ished quantity
1	10	0.002	429.55	2.62	7.12	117.88	0.79
2	10	0.011	434.11	2.90	8.79	118.98	4.36
3	10	0.020	438.70	3.24	10.97	119.58	7.97
4	15	0.002	431.61	1.99	5.49	121.56	0.81
5	25	0.002	433.42	1.36	3.76	125.08	0.83

points may be determined by solving the dynamic programming equations:

$$TC_k = Min\{TC_j + W(j, p^*, k)\} \quad (16)$$

$$TC_0 = 0 \quad (17)$$

The forward recursive procedure is used to determine the minimal total cost over planning horizon H. To solve the model, a dynamic programming algorithm is proposed by Chen (1998).

3. DATA AND RESULTS

A numerical example is presented to show the effect of different demand distributions, urgency and perishability of a relief item. The values used in the numerical example are useful for illustration purposes but may not match values that might be estimated by the ultimate users of the model.

Parameters of demand function are a_o and a_1. The values of a_o and a_1 are set to 25 and 0.1, respectively. The planning horizon (H) is assumed to be fixed and set to a value of 50. Thus total demand in the planning horizon is 248.3 unit. Relief item perishes due to several reasons including poor preserving facilities after a disaster. Perish rate (θ) is set to 0.002. It is natural to assume that the shortage cost per unit (P_0) is higher than the holding cost per unit (H_0). Note that the shortage cost per unit does not depend on urgency parameters. However, realized shortage cost is dependent on urgency parameter. Here, P_0 and H_0 are set to 1 and 0.3 respectively. The operational cost per unit (C_0) is set between P_0 and H_0. The value of C_0 is equal to 0.5. The parameters for urgency (γ and μ) are set to 10 and 0.08, respectively. The value of gamma (γ) changes in different scenarios. Urgency is used to generate the realized shortage cost, policy makers can regulate the parameters of urgency. If γ is equal to zero, the value of urgency becomes one. Therefore, the realized shortage cost and the shortage cost become identical values. Note that, different scenarios are created by altering the value of perish rate (θ) and gamma (γ). Table 1 presents a scenario description and the computational results for different scenarios. The perish rates for scenarios 1 to 3 are varied and the urgency parameter (γ) value is kept fixed. In contrast, scenarios 1, 4, and 5 have characteristics of different urgency parameter (γ) values for a fixed perish rate (θ). Finally, the ordering fixed cost (A_0) is equal to 30.

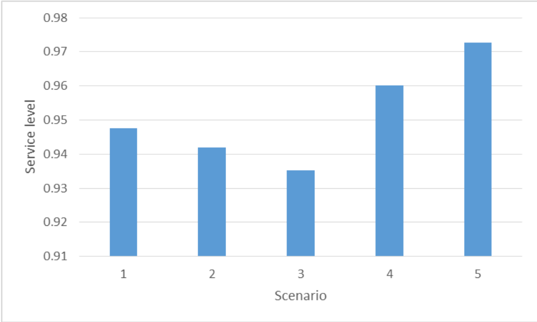

Fig.1 Service levels in different scenarios

According to Table 1, total cost and out of stock are the highest for higher perish rate. A comparison between scenarios 1, 4, and 5 reveals that the total out of stock duration is shorter and the quantity perished is larger for higher urgency values. On the other hand, total out of stock duration is longer for higher perish rate (in scenario 1, 2, and 3). Therefore, the model rationally responds to a changing environment.

Now, we define service level (SL) as in eq (18):

$$SL = 1 - \frac{OutofStockDuration}{PlanningHorizon} \quad \textbf{(18)}$$

Fig.1 presents the computed SL for each scenario. The service level changes with the combined effects of γ and θ. The service level becomes higher for larger gamma value. It shows that policy maker can impose larger value of gamma for improving service level.

To elaborate detailed results, we present the outcomes of scenario 1 in Table 2. It has nine (9) cycles. The start and the end points for each cycle period are tabulated in the second column in Table 2. Cycle lengths are different during the planning horizon. Generally length of cycle period become longer sequentially. Table 2 shows the relief items' arrival time (p^*) after the start of each cycle. The replenishment point in each cycle is also tabulated in the third column.

Table 2 Results of scenario 1

Cycle	Cycle period [j,k]	p^*	Holding cost	Shortage cost	Ordered quantity	perished quantity
1	[0,2]	0.05	12.49	0.34	45.40	0.083
2	[2,4]	2.06	10.15	0.31	37.17	0.068
3	[4,6]	4.06	8.23	0.29	30.43	0.055
4	[6,9]	6.11	14.11	0.57	35.65	0.094
5	[9,12]	9.13	10.27	0.51	26.41	0.068
6	[12,16]	12.20	12.48	0.73	24.91	0.083
7	[16,21]	16.31	11.95	0.87	19.94	0.080
8	[21,27]	21.46	9.43	0.89	13.88	0.063
9	[27,50]	28.25	28.77	2.62	15.31	0.19
Total			117.88	7.13	249.1	0.781

Generally, a longer cycle length leads to an item that perishes faster. Table 3 provides the holding cost, the shortage cost, the ordered quantity, and the perished quantity for each cycle.

To observe the effect of different declining demand and urgency, we conduct a sensitivity analysis and compare the results from four situations. Table 3 presents the properties of the four types of situations. Type A represents the exponential declining demand and urgency are included in the computation. Similarly Type B also include urgency but demand declines linearly. Type C and D does not include urgency in the computation (i.e,*γ=0*). Herewith, demand function in Type C is exponential declining function and that in Type D is linear declining function. The differences of total demand between linear and exponential distribution is 0.5%. Note that perish rate is set to 0.08. The planning horizon and other

Table 3 Properties of four types of situations

Type	Properties	Mathematical explanation
A	Exponential declining demand and urgency incorporated	$D(t)=25e^{-0.1t}$, θ =0.08, and γ =2
B	Linear declining demand and urgency incorporated	$D(t)=10-0.2t$, θ =0.08, and γ =2
C	Exponential declining demand and no urgency	$D(t)=25e^{-0.1t}$, θ =0.08, but γ =0
D	Linear declining demand and no urgency	$D(t)=10-0.2t$, θ =0.08, and γ =0

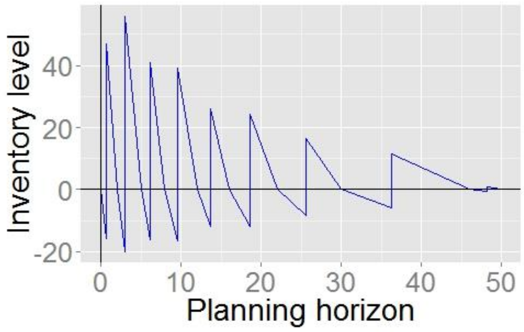

Fig.2 Inventory level for Type A

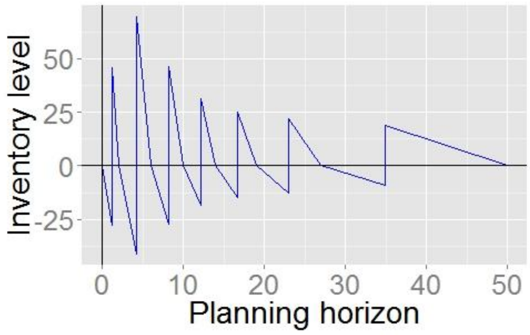

Fig.3 Inventory level for Type C

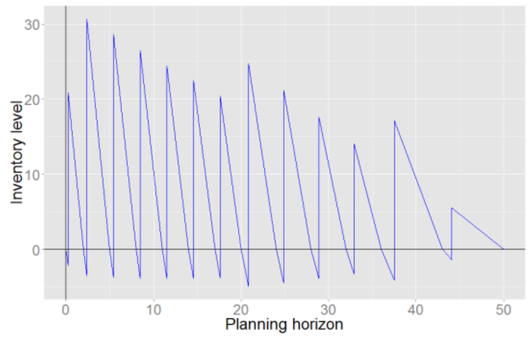

Fig.4 Inventory level for Type B

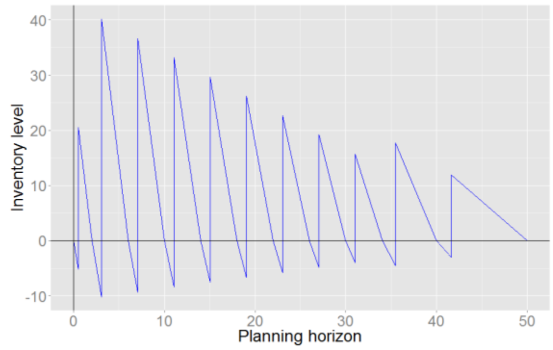

Fig.5 Inventory level for Type D

parameter values are kept same as before.

Fron Fig. 2 to Fig. 5 illustrate the results of the four situations. An analysis of the four types of situations shows that the first and the last cycles have irregular shapes. The irregularity of the first cycle occurred because the value of the initial cost (TC_0) is zero. The last cycle is irregular because the algorithm is based on a forward recursive procedure for a fixed time horizon. The model brings a zero inventory level after satisfying all of the demand in the last cycle.

Among the four types of situations, the total number of replenishment points is smaller while demand distribution is exponentially distributed. In the comparison between Type A and C, the total number of replenishment point is larger when urgency is incorporated. Similarly Type B has more replenishment points than Type D.

For linear demand declining function (for Type B and D), the quantity of delayed satisfied demand shows opposite trend. The delayed satisfied demand increases gradually while urgency is considered (Type C). However, the satisfied demand decreases while urgency is not incorporated (Type D). On the other hand, for exponential declining demand function, the quantity of delayed satisfied demand follows similar trend (Type A and C).

From the analysis, it is found that declining demand properties, urgency and perishability have significant influence on decision making on relief ordering policy. It is also found that there is tradeoff between perish rate and post disaster cost. Therefore, policy makers can invest money for improving facilities such as perish rate become smaller. It will be helpful for reducing the total cost by decreasing perished quantity. Since available relief after a disaster is limited, it is essential for adopting policies for decreasing perish rate. In addition, policy maker has sufficient control on urgency value to meet sufficient service level in relief distribution.

4. CONCLUSION

This proposed dynamic programming model incorporated urgency and perishability for decision making on relief ordering. The replenishment intervals vary for different parameters however there is a trend in variation. The service level also changes between replenishment cycles. As a result, the model generates a better solution than other optimization models with fixed order intervals and/or fixed service level. The model is compared for two demand distributions and different urgency parameter (gamma) values.

The results of the study lead to the following conclusions. Urgency and perishability has significant influence on relief ordering. These issues affect the system service level. A decision maker must plan for relief ordering considering the possible wastage due to perishability.

Although the model presented is flexible, the main restrictions to its practical implementation are twofold: continuous review of the stock is assumed and the shape of the urgency function is not known with certainty. Fortunately, organizations are making efforts to collect relief operational data, allowing for a true urgency function to be generated in the future. Herewith, we have not incorporated the disposal cost for perished item.

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