# Planning Model of Optimal Modal-Mix in Intercity Passenger Transportation

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**Abstract.** Environmentally sustainable transportation becomes an important issue as well for intercity passenger transportation, where modal shifting from energy consuming airline and bus service to energy efficient high speed railway is the most feasible measure. But due to the less flexibility and fixed to locations of railway improvements, strategic redistribution of network-wide demand onto the improving rail lines is required. This paper presents an optimal modal-mix planning model in intercity passenger transportation, which aims to design a modal mix network of least  $CO_2$  emissions and less total travel time, as well as less intermediate transfer cost, considering feasibility and economical sustainability of the service frequency. The proposed model is formulated as a mixed integer linear programming model, which can be numerically solved by general solver programs.

Keywords: Modal-mix, Intercity Transportation, Network Design, sustainability.

#### **1** Introduction

Environmentally sustainable transportation concept was first proposed by OECD as urban transportation and urban planning context, but recently discussion were going on intercity passenger transportation field as well, for example EU's idea of high speed railway service substitution of shorter feeder airline service. Considering large difference of energy intensity and unit  $CO_2$  emissions, we can say that modal shifting from energy consuming airline and bus service to energy efficient high speed railway is the most feasible measure.

We want to check such possibility of strategic redistribution of network-wide demand onto the improved rail lines, and resulted reduction of energy-use and  $CO_2$  emissions. This paper presents an optimal modal-mix planning model in intercity passenger transportation, which aims to design a modal mix network of least  $CO_2$  emissions and less total travel time, as well as less intermediate transfer cost, considering feasibility and economical sustainability of the service frequency.

### 2 Outline of Planning Problem

As the objective of network design problem, total travel time, total generalized cost, asset cost, as well as total  $CO_2$  emissions can be considered. In order to formulate a multi-criteria optimization problem, a single objective function is often synthesized by weight parameters for each single component objective. Recently, minimax problem which minimize the largest unsatisfactory level in

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several objective component was formulated into linear minimization function.

This paper considers the problem of finding multi-modal route traffic flow shares for given OD traffics and necessary rail, bus and air frequencies on each link which minimize total  $CO_2$  emissions. In order to avoid too long detouring of passengers and too many transfers between different modes, we consider total travel time and total transfer cost of passengers as other objective components to be minimized. In order to secure the feasibility and economical sustainability of the service frequency on each link, we set the frequency providing enough capacity for the assigned traffic flow on the link. Furthermore, the assigned passenger numbers on that link must be larger than the required passenger number to sustain the frequency level.

## **3** Model Formulation

#### 3.1 Variables and Parameters

In our network design, each city is represented by a node n  $(n \in N)$  and connecting arcs between nodes i, j through different modes m  $(m \in M)$  are indicated by  $(i, j) \times m \in A$ . In order to express transit connection between modes explicitly, each node is divided into arrival node by mode m as  $n_{-} \times m$  and departure node by mode m' as  $n_{+} \times m'$ , and transit arc between them is indicated by  $(m,m') \times n$ . Also, amount of OD traffic between zones  $(k,l) \in K \times K$  is given by  $T_{kl}$ . Endogenous and exogenous variables are explained in Table 1.

#### 3.2 Objective Function and Minimax Problem

In this model, we pick up total passenger travel time P, total passenger transit cost Q and total CO<sub>2</sub> emissions associated with transport operations V as the objective components to be minimized:

$$P = \sum_{i} \sum_{j} \sum_{m} t^{m}_{ij} \sum_{k} X^{km}_{ij} \quad .$$
<sup>(1)</sup>

Variable	Explanation
$X_{ij}^{\ km}$	Traffic amount on an arc originated from node $k$ by mode $m$
$Y_n^{kmm'}$	Amount of transit passengers from mode $m$ to $m$ ` at node $n$
$B_k^m$	OD trips originated from node $k$ using mode $m$
$A_n^{km}$	OD trips between k and $n$ using mode $m$
$Z^m_{ij}$ , $F^m_{ij}$	Existence of service and frequency on an arc for mode <i>m</i>
$t_{ij}^{m}$	Travel time on an arc for mode <i>m</i>

Table.1 Endogenous and exogenous variables.

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$ au_n^{mm'}$	Transit cost between modes $m$ and $m$
$h^m$ , $g^m$	Seat capacity and max. operable frequency of mode <i>m</i>
$c_{ii}^m$	CO <sub>2</sub> emissions per one service/flight on an arc
$d_{ij}^m$ and $e_{ij}^m$	Fixed and variable cost of maintaining service on an arc

$$Q = \sum_{n} \sum_{m} \sum_{m'} \tau_n^{mm'} \sum_k Y_n^{kmm'}$$
(2)

$$V = \sum_{i} \sum_{j} \sum_{m} c_{ij}^{m} F_{ij}^{m}.$$
 (3)

These 3 objective components are not suitable for integration because their scales are different. Therefore, we set new p, q, v values below to be scaled down between 0 and 1 by using ideal values  $P^*$ ,  $Q^*$ ,  $V^*$  and evaluation values  $P_0$ ,  $Q_0$ ,  $V_0$ .

$$p = \frac{P - P^*}{P_0 - P^*}, q = \frac{Q - Q^*}{Q_0 - Q^*}, v = \frac{V - V^*}{V_0 - V^*} \quad .$$
<sup>(4)</sup>

Here, in order to optimize all of three objectives, we consider minimax problem by the introduction of  $\lambda$  representing the most inferior objective, and sufficiently small positive constant  $\varepsilon$ , as follows:

$$\min_{X,Y,B,A,Z,F} \lambda + \varepsilon (p+q+v), \quad p \le \lambda, \ q \le \lambda, \ v \le \lambda$$
<sup>(5)</sup>

#### 3.3 Constraints

First, we describe the conditions for the preservation of traffic amount. Regarding to the arriving traffic at each node n, the following two equations are satisfied:

$$\sum_{i\in N^{-}(n)} X_{in}^{km} = A_n^{km} + \sum_{m'\in M} Y_n^{kmm'} \ \forall n\in N, \forall k\in K, \forall m\in M$$
<sup>(6)</sup>

$$\sum_{m} A_{n}^{km} = T_{kn} \ \forall n \in N, \forall k \in K$$
<sup>(7)</sup>

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Similarly, regarding the passengers departing from each node n, following two equations are satisfied:

$$B_n^m + \sum_{m' \in M} Y_n^{km'm} = \sum_{j \in N^+(n)} X_{nj}^{km} \quad \forall n \in N, \forall k \in K, \forall m \in M$$
<sup>(8)</sup>

$$\sum_{l \in K} T_{nl} = \sum_{m \in M} B_n^m \ \forall n \in K$$
<sup>(9)</sup>

Next, the constraints about the frequency set up will be described by Eq. (10)~(13)

$$F_{ij}^{m} \leq g^{m} Z_{ij}^{m} \ \forall (i,j) \times m \in A.$$
<sup>(10)</sup>

$$\sum_{i\in N^-(n)} F_{in}^m = \sum_{j\in N^+(n)} F_{nj}^m \quad \forall n\in N, \forall m\in M .$$
<sup>(11)</sup>

$$\sum_{k} X_{ij}^{km} \le h^m F_{ij}^m \quad \forall (i,j) \times m \in A.$$
<sup>(12)</sup>

$$\sum_{k} X_{ij}^{km} \ge d_{ij}^{m} Z_{ij}^{m} + e_{ij}^{m} F_{ij}^{m} \quad \forall (i,j) \times m \in A.$$
<sup>(13)</sup>

Finally, followings are added as the domain of variables.

$$X_{ij}^{km} \ge 0, Y_n^{kmm'} \ge 0, B_k^m \ge 0, A_n^{km} \ge 0.$$
 (14)

$$Z_{ii}^{m} = \{0,1\}, F_{ii}^{m} \ge 0.$$
<sup>(15)</sup>

As mentioned above, the problem which takes Eq. (5) as objective function and takes Eq. (1)~(4) and Eq. (6)~(15) as constraints turns into a mixed linear programming problem containing a small number of 0-1 variable ( $Z_{ij}^m$ ). Therefore, proposed mixed integer linear programming model can be numerically solved by general mathematical software packages.

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#### 4 Numerical Example and Conclusion

The proposed model has been applied to a small network consists of 6 nodes and 20 links of 3 different modes (railway, bus and airline) as shown in Figure-1.

Objective	Total Travel Time	Total Transfer Cost	Total CO <sub>2</sub> Emiss.
Min. Travel Time (P)	3.890.636	18.240	113.590
Min. Transfer Cost (Q)	7.765.335	0	30.333
Min. CO2 Emission (V)	6.004.489	0	22.558
Optimal Solution (PQV)	5.413.639	0	35.781

 Table 2. Resulting Objective Values



Figure.1 Sample Network

We used LPSolve package for solving the equations by the methodology explained in section 3.2 using above data. Resulting P, Q, V values are shown in Table-2. Figure-2 illustrates optimal network shape with link frequencies and passenger numbers on links. Considering the low unit emission of  $CO_2$  by rail comparing to the air and bus, the result network is mainly consisted by rail links. However, for between city 1 and 6, where trip time by rail via city 3 is 308 minutes, too larger, direct air service of 100 minutes is provided. Similarly, bus service is provided on link 4-6, where trip time of rail is much longer than bus. For link 4-5, where bus service is slightly faster than rail, co-existence of bus and rail is observed. As described above, the proposed model successfully give a best-mix design of inter-city modal-mix network.

In conclusion, we have presented an optimal modal-mix planning model to design a modal mix network of least  $CO_2$  emissions from the viewpoint of transport operators. We applied the model on a sample network successfully using mixed linear integer programing tools. Resulting optimal network shape and required frequencies for sustainable operation for given OD demand were also presented. This paper

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**Figure.2** Resulting network with link frequencies and passenger numbers on links provides an upper-level model to consider operators behavior of the network design problem. For the future study, passengers' route choice behavior should be modeled as the lower-level of the network design problem.

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