Influence on Rank-size Rule for Distribution Centers under the Innovations in Logistics

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Abstract:

This study aims to clarify the location characteristic of the optimal distribution centers obtained by the joint inventory / distribution problem. Simulation analysis under different sets of parameters concerning logistics and transportation characteristics would give different / indifferent geographic distributions of DC location and inventory. The calculated locations of DC are analyzed through the rank-size coefficient, which indicates that geographical distribution of DC location is more scattered than that expected by Zipf's law. If a pressure from demand side for stock-out, fewer number of DCs with larger stocks opt to locate at lower land rent zones. Discount of expressway toll makes no particular influence on the rank-size coefficient.

Keywords: Delivery logistics, Inventory, Location of distribution centers

1. Introduction

Drastic innovations in information technologies (IT) started in 1990's haves pushed further innovations in manufacturing, transportation, and retail industries. Coincide with the shift from "product-out" to "market-in" principle in marketing science, Supply-Chain-Management (SCM) becomes a focal issue in logistics field instead of "Just-in-time" principle managing intermediate goods flow within the supply side (Hameri and Paatera, 2005). SCM requires an efficient integration among the above sectors in order to enable more value-added service. A famous business model based on SCM is e-commerce such as "Built to order" system from the end users by DELL Computer Inc. (Gunasekaran and Ngai, 2005). "Built to order" system gives novel channel between a firm and customers such that a customer requests to make ready-made personal computer "personalized" through the web site, the order immediately let factories start to assemble the requested parts, and then the "personal" computer built in the request is delivered in a few days. In this system, appropriate information sharing among the sectors is of importance.

After the appearing of e-commerce, penetration of IT among our economy has rapidly progressed. Lassere (2004) pointed out that growth of e-commerce stimulates more improvement in logistics service especially for delivery sector following to customer's needs for convenient service. Therefore, we can find a positive feedback among demand and supply side under advanced IT, such that the more improvement in delivery service on average is done by supply side, the more requirements on convenient delivery occurs from demand side. Hesse and Rodorigue (2004) showed a figure about longitudinal change in logistics from 1960s to 2000s. Average cycle time required to produce a completed good from its materials is decreased about one eighth of 1960s in 2000s, while decrease in transportation and in inventory cost are moderate such as two third of 1960s in 2000s. They pointed out that average cycle time is effectively decreased by enforcing efficient management in supply side, while some of cost in transportation and inventory can not be decreased in order to keep demand side service higher.

A trade-off in terms of cost between goods distribution and inventory management, that is, less inventory at distribution center (DC) is incompatible with quick distribution by placing DC close to customers with large inventory, is important focal issue to understand logistic network design. The joint inventory / distribution problem has been intensively studied in applied operations research (ReVelle and Laporte, 1998). Jayaraman and Pirkul (2001) formulated multi-plant, multi-product distribution model with multiple-echelon to simultaneously determine the location of plant and DCs, with the quantity of raw, intermediate and final goods flows among plant, DCs and customers. A joint inventory / distribution problem is further expanded into a joint inventory / routing (around customers) problem to find optimal routing around customers (Agezzaf, E. *et al*,, 2006).

Most of these previous studies in operations research contribute to obtain an optimal solution efficiently, however, researchers merely have deductive viewpoint on geographical characteristics of the sets of optimal DC location and inventory. Okumura and Tsukai (2003) applied joint inventory / delivery model with a given parameter set to various whole Japan transport networks, and then found the robustness of distribution center locations by intuitive inspections, but such naive inspections have a limitation to generalize the obtained characteristics over the different conditions. Therefore in this study, the change of the rank-size distribution of DC characteristics such as operated demand, and prepared stocks will be tested by the rank-size coefficient obtained under various sets of parameters.

Studies on rank-size distribution has been attracted strong interests of geographers. Auerbach (1919) found that city size distribution could be closely approximated by Pareto distribution. Here, rank of cities is numbered from largest (rank 1) to smallest (rank N) to get the rank p for city size of $S(p)$. Then eq.(1) would describe the relationship between p and $S(p)$.

$$\log S(p) = \log A - \theta \log p \quad (1)$$

where A and θ (rank-size coefficient) are parameters. Zipf (1949) reported that city size follow a special form of the distribution where $\theta = 1$, which is called Zipf's law. Soo (2005) reported cross-country difference on Zipf's law by empirically estimating the rank-size coefficient. Nitsch (2005) summarized conventional studies by meta-analysis on 29 studies on 515 rank-size coefficients, giving that supportive result for Zipf's law.

In this study, we aim to clarify location characteristics of distribution centers obtained as the optimal location by the joint inventory / distribution problem. Simulation analysis under different sets of parameters reflecting customers and geographic characteristics would give different / indifferent geographic characteristics of DC location and inventory. The location characteristics of these simulation results are analyzed through the rank-size coefficients estimated from the located DC characteristics, in terms of operated demand and prepared stock size. This approach will give more proper understanding about the change in geographic advantages against supply and demand side structural change.

The sections are organized as follows. Sec.2 explains two echelon inventory system as joint inventory / distribution problem, and shows how it is formulated as an optimization problem. Sec.3 is for calculation procedure for two echelon inventory system, and for the estimation of rank-size coefficient. Sec.4 demonstrates the proposed model applied for truck sales system in Japan. Sec.5 summarizes the outputs and further issues to be studied.

2. Two echelon inventory system

2.1 Replenish interval and inventory

In this study, we adopt joint inventory / distribution model considering stochastic demand proposed by Nozick and Turnquist (2001). The proposed model has a two-echelon distribution network formation problem and endogenized optimal inventory allocation between a central logistic center (called as "plant") and several number of "distribution center (DC)s" under stochastic demand, as illustrated in Fig. 1. Since the basic configuration of the model follows our previous study, we briefly

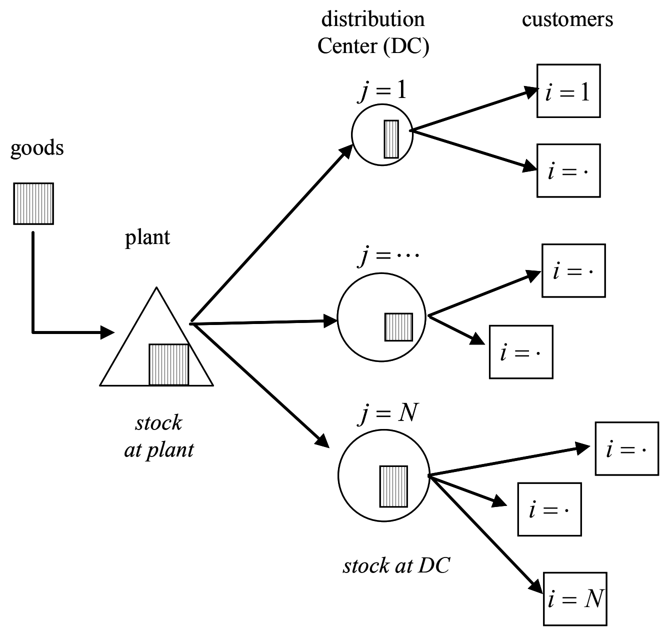

Figure 1 Two-echelon goods distribution network

shows the model used in this study, see details in Okumura and Tsukai (2003).

Although many manufacturing firms begin to manage total supply chains including parts and material supply process, in reality, their final assembly plants can not be easily relocated. Moreover, many firms try to make the market chain simpler and reduce the number of echelons. Therefore, we concentrate our analysis on market chain from a plant to customers, so then the location of plant (i.e. transportation cost between plant and DCs) is not considered. The proposed two echelon system is composed by one "plant" (indicated by $j = 0$), and many DCs (indicated by $j = 1,\cdots,N$) providing the distribution service to the retail outlets (indicated by $i = 1,\cdots,I$) locating all over the country, replying to the orders. From final manufacturing factories, finished products are sent to the "plant", once in a predetermined interval μ_0 to make the plant storage full (s_0). From the plant, several numbers of products are sent in a given interval μ_1, in order to replenish of DC stock (s_j). Each DC (indicated by j) has a full stock of s_j just after the replenishment, and sends one product when it receives an order from a retail outlets under its supervision area (i for $Y_{ij} = 1$). Orders from each retail outlet are assumed to follow mutually independent Poisson distribution with given arrival rate (λ_i). If Y_{ij} be the proportion of demands at retail outlet i supplied by DC j, the aggregated order arrival at DC j is also given by a Poisson distribution, whose arrival rate Λ_j is given by

$$\Lambda_j = \sum_{i=1}^{I} Y_{ij}\lambda_i \tag{2}$$

If the number of orders in the given replenishment interval (μ_1) exceeds the storage size (s_j), stock-out occurs and makes the customer wait until the next replenishment. Possibly some customers prefer canceling to waiting, then make the firm loss of profit. Such loss is evaluated as parameter α .

The probability of DC stock-out $r(s_j)$ is given by the following, when m_j is number of orders at DC j during the replenishment interval μ_1 .

$$r(s_j) = \text{Prob}\left(m_j > s_j\right) = \sum_{m_j = s_j+1}^{\infty} \frac{\exp(-\Lambda_j \mu_1)(\Lambda_j \mu_1)^{m_j}}{m_j} . \tag{3}$$

The total demand at the plant is also a Poisson process with mean arrival rate.

$$\Lambda_0 = \sum_{j=1}^{N} \Lambda_j = \sum_{i=1}^{I} \lambda_i \tag{4}$$

The stock-out probability $r(s_0)$ at the plant with capacity s_0 under the replenishment interval μ_0 is given by the similar equation with eq.(3), such as

$$r(s_0) = \text{Prob}\left(m_0 > s_0\right) = \sum_{m_0 = s_0+1}^{\infty} \frac{\exp(-\Lambda_0 \mu_0)(\Lambda_0 \mu_0)^{m_0}}{m_0} \tag{5}$$

where m_0 is number of orders at the plant during the replenishment interval μ_0 . There is no direct effect of plant stock-out on customers, as long as stock remains at the DCs. However, once stock-out is happened at the plant and backorders accumulate at the plant, the succeeding replenishment to DCs must be postponed. The average additional waiting time at the DCs is given by the expected number of backorders at the plant, divided by the plant demand rate, Λ_0 , according to the Little's Law.

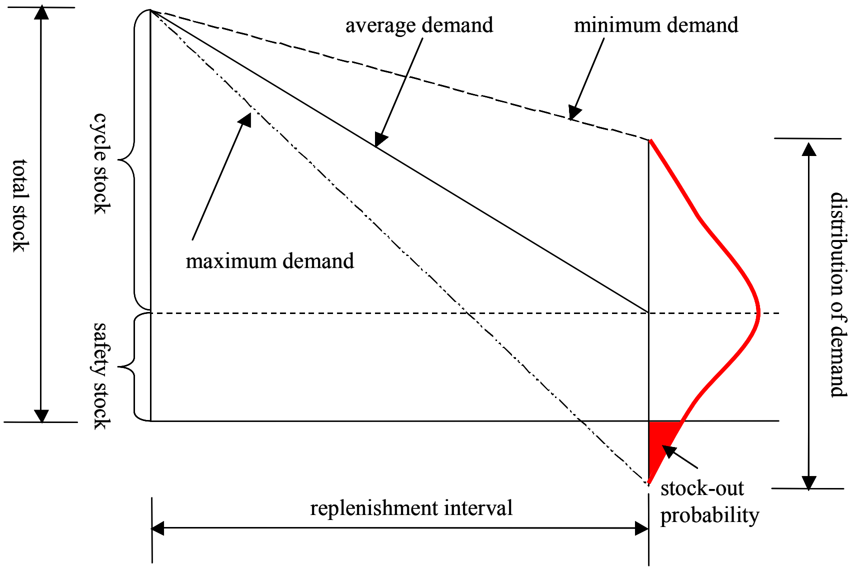

Figure 2 Dynamics of the stock at distribution center during replenishment interval

$$W_0 = \left\{ \sum_{m_0 = s_0 + 1}^{\infty} (m_0 - s_0) \frac{\exp(-\Lambda_0 \mu_0)(\Lambda_0 \mu_0)^{m_0}}{m_0} \right\} \Big/ \Lambda_0 \quad (6)$$

The replenishment interval from the plant to the DCs μ_1 in eq.(3) is replaced by the expected replenishment time as,

$$\mu_1' = \mu_1 + W_0 \quad (7)$$

This replenishment postponing violates the assumption of independency between the demand and replenishment processes, but the differences are considered to be minor (Diks *et al.*, 1996).

In case of simultaneous stock-outs at the plant and at some DC, the customer must wait longer, and the expected profit loss becomes larger than the case of stock-out in DC only. The expected loss is indicated by a given parameter β, which is naturally larger than α for stock-out at DC only.

In order to avoid such stock-out losses, larger number of products than the average demand must be stored at DCs and at the plant. In logistic theory, the stock for average demands during the replenishment time is called as "cycle stock," while additional stock over that cycle stock is called as "safety stock." In this model, we assume that safety stock is non-negative, then,

$$s_0 \geq \mu_0 \Lambda_0, \; s_j \geq \mu_1 \Lambda_j \quad (8)$$

Fig. 2 illustrates the typical dynamics of the stock at one DC during replenishment interval. When excess demand during the interval becomes larger than the safety stock, DC stock-out occurs. Without safety stock, the cycle stock can cover the fluctuating demand with just 50 % of probability. As more safety stock is prepared, stock-out probability becomes smaller. In order to prepare the stock at the plant or at each DC site, corresponding cost is required.

Assume that for each site, total inventory cost C_j can be given as linear function of stock size s_j, with certain fixed cost f_j. Let h_j be unit cost for storage capacity, then,

$$C_j(s_j) = f_j + h_j s_j \quad (9)$$

Contrast to the original formulation, we permit the heterogeneity of unit cost h_j, as well as the fixed cost f_j according to the location of DC. In our analysis, those costs are given reflecting the land price of each location.

Similarly, at the plant, storage cost C_0 is given by the following linear function of storage size s_0;

$$C_0(s_0) = f_0 + h_0 s_0 \tag{10}$$

2.2 Optimal stock allocation model

In order to know the most efficient level of safety stocks at the plant and the DCs, the following cost, which consists of expected stock-out penalty and inventory costs, must be minimized, with non-negative safety stock conditions eq.(8).

$$\min_{s_j, s_0} \quad \alpha[1 - r_0(s_0)]\sum_{j=1}^{N} \Lambda_j r_j(s_j) + \beta r_0(s_0)\sum_{j=1}^{N} \Lambda_j r_j(s_j) + \sum_{j=1}^{N} h_j s_j + h_0 s_0 \tag{11}$$

$$s_0 \geq \mu_0 \Lambda_0, \ s_j \geq \mu_1 \Lambda_j \tag{8}$$

2.3 Optimal DC location selection

To search the efficient number of the DCs; N and location of each DC, we can utilize optimal facility location problem minimizing the total cost composed by the location cost and transportation cost, as formulated in the field of operations research. We take the following assumptions in order to simplify the location problem.

1) Consider a firm whose customers are locating all over the country.
2) Products are conveyed one way from the plant to DCs, and from each DC to retail outlets locations supervised by the DC by trucks.
3) Transportation cost between the plant and DCs is negligible, because that transportation in large lot size require relatively small unit cost comparing to the more frequent lower transportations in smaller lot size from DC to the retail outlets.
4) The fixed location cost f_j and unit storage cost h_j are given in proportional with land price of the location j.
5) The location of the plant is exogenously given.

Optimal facility location problem to give the number of DCs and the locations can be formulated as follows, when k be the candidate location set for DCs.

$$\min_{X_k, Y_{ik}} \quad Z_{IP}^{p} \sum_{k=1}^{K} C_k X_k + \sum_{k=1}^{K}\sum_{i=1}^{I} g\lambda_i d_{ik} Y_{ik} \tag{12}$$

$$\text{subject to} \qquad X_k \in \{0,1\} \qquad \forall k \in K \tag{13}$$

$$\sum_{k=1}^{N} Y_{ik} = 1 \qquad \forall i \in I \tag{14}$$

$$Y_{ik} \leq X_k \qquad \forall k \in K, \ \forall i \in I \tag{15}$$

where, X_k is integer variable indicating the existence of DC at location $k \in K$, Y_{ik} is the proportion of demand in i supervised by DC k, C_k is location cost of DC at location k, g is unit time period, and d_{ik} is unit transportation cost between location k to i. Constraint (14) is a condition for DCs to cover all $i \in I$. Constraint (15) is a consistency between X_k and Y_{ik}, if customer in i can not be assigned to k without DC (eliminating $Y_{ik} = 1$ when $X_k = 0$).

Because Y_{ik} is also binary due to the consistency condition of (15) and binary definition (13) of X_k, this problem is integer programming problem (IP).

3. Calculation procedure for the models

3. 1 Integrated model and optimization procedure

Combining the two minimization problems (11) and (12), we can get optimal number of DCs, location and stock size at each DCs, as well as stock size at the plant.

At first, exogenous parameters, $\alpha, \beta, \mu_1, \mu_0, \lambda_i, f_j, h_j, f_0, h_0$ and d_{ik} are given. Solution of the second location problem minimizing eq.(12), Y_{ik} gives the demand arrival rate of each DC Λ_j through eq.(2) as the input for the first stock assignment problem minimizing eq.(11).

The first model is a non-linear problem whose solution space has dimension of $N+1$ over control variables s_0 and $s_j s$. However, there are no interactions between the stock capacities of the different DCs in eq.(11), the optimization problem is separable and monotonic for each s_j. The following procedure can be used considering that s_0 and $s_j s$ are integer variables satisfying eq.(8).

1) Set s_0 be the smallest integer value no less than the cycle stock at the plant, $\mu_0 \Lambda_0$.
2) For each DC j, set s_j be the smallest integer value exceeding $\mu_1 \Lambda_j$, calculate the value of the objective function eq.(11).
3) Increase s_j one by one until the total cost begins to increase. Keep s_j^* as the candidate for the optimal solution.
4) After all DC stocks are determined, the value of optimal function for s_0 and s_j^* is calculated and kept for candidate solution.
5) Unless the function value increase, add one to s_0 and iterate the steps from step 2), above.

Using the solution of the first problem, s_j, we refresh the location cost for each DC through eq.(9), which is required for the location problem. However, eq.(9) give the location cost only for the sites where DC locates in the present situation.

The original study of Nozick and Turnquist (2001), proposed two alternate ways to give the stock capacity where DC is not locating at present. One way is to determine a critical stock-out level and know the total stock required in total system (plant and all DCs). If we divide that value by N after subtraction of s_0, required stock level is estimated. The other way is to give the average stock of the present DCs for all potential locations. In their work, they neglect any differences in unit stock cost h_j by location, those two ways give the similar results. But if we introduce the heterogeneous stock cost h_j in each location, both of their approximations of stock level give a trouble in conversion of iterative process. We take therefore, a different way to give the stock for potential DC locations, which is compatible when optimal solution is met. The way is to assume that if a potential location is selected, then such new DC takes over the function of the DC now responsible for that location, instead. Then, the same amount of stock must be prepared. This assumption is formulated as following,

$$s_k^* = s_i, \quad \text{such that } Y_{kj} = 1 \tag{16}$$

Then, we can set the location cost in the location problem is given as,

$$C_k = \begin{cases} f_k + h_k s_k & \text{if DC locates at } k \\ f_k + h_k s_j & \text{unless DC locates at k} \end{cases} \tag{17}$$

With this procedure, all parameters of the optimal location problem (12) are fixed and can be solved by an appropriate algorithm for non-capacitated facility location problem.

If binary condition (13) is relieved to positive real, we get a linear programming (LP) and simplex method is applicable to get the optimal solution Z_{LP}^p (Campbell, 1990). Due to a strength of constraint for solution space, Z_{IP}^p is not less than Z_{LP}^p and equal sign only appears when optimal LP solution is integer. However, simplex method needs a long calculation time for the problem with many constraints. Our model includes N (number of DC candidates) $\times I$ (number of demand locations) constraints, but the constraints matrix is very sparse. Such problem can not be effectively solved even by modern LP or interior point method. Another popular algorithm for IP is branch and bound method, which is an enumeration method using lower bound information of objective function. This procedure makes sub-problems by setting restrictions on some locating candidates k (i.e. $X_k = 1$ or $X_k = 0$ for some k), which is called as "branch," and estimate the lower bound of the branch k. If the lower bound of the branch k is inferior to another branch that is already estimated, we can terminate the branch k and move to further branch, which is called as "bound." Therefore, the efficiency of the branch and bound critically depends on the accuracy of lower bound and calculation time for sub-problems. Erlenkotter (1976) proposed a dual ascent / dual adjustment procedure to optimize the dual problem shown in objective function Z_{IP}^p; (12) and constraints; from (13) to (15), which searches the optimal solution within the integer space. We use this procedure because this algorithm for the sub-problem is satisfactory with accuracy and quickness.

3.2 Estimation of rank-size coefficient

If optimal DC location is obtained by applying the procedure shown in 3.1, rank-size coefficients θ with constant term $\log A$ in eq.(1) for DC arrival demand rates Λ_j, for DC stock size s_j are estimated by OLS. Note that previous studies claim the bias in OLS estimates for rank-size coefficient, because the error term does not follow normal distribution. Nishiyama, *et al.* (2006) reported that OLS estimates of θ; $\hat{\theta}^{OLS}$ have downward bias, and it is also hold for OLS estimates of σ_θ; $\hat{\sigma}_\theta^{OLS}$, respectively. Therefore, a null hypothesis $H_0 : \theta = -1$ tends to be rejected. In order to avoid this bias, for example, maximum likelihood estimator with assuming Pareto distribution to error term (Hill, 1975) is used in empirical study (Soo, 2005). However, our purpose of this study lies to make comparison under several parameter settings, and the rank-size coefficients are used for a statistics to summarize geographical feature of DC location and inventory. The statistical test between the estimated parameter is not made, which leaves as a further research issue.

4. Results in rank-size coefficient for the characteristics of distribution center

4.1 Case Setting

We consider the distribution from one domestic plant to the 207 regions all over Japan, through highway network in year 2000. Demand arrival rate in each region is given by allocating the annual domestic truck sales in year 1995 (177,264 vehicles / year) into each region with proportion to the number of the registered trucks there.

Inter-regional transportation cost d_{ij} is given by the generalized cost including the expressway fare for truck and time value (3,000 yen / hr) of driving time between the regions through the shortest time path based on expressway, national and local primary road network. Since the target network is inter regional, we can neglect the congestion (transportation time is flow independent). We calculate it by using GIS function for the network in year 2000.

DCs are considered to be locatable at any of 207 regions. Both fixed location cost f_j and unit stock cost h_j are set reflecting the land price level in each location. We assume that each DC requires fixed area for office ($100\,m^2$) plus unit parking space ($30\,m^2$) times the stock capacity, s_k. Assume the business length of each DC be ten years. We consider the firm purchase the land for DC and that cost must be returned by flat payment for the years, with 4% of interest rate. Therefore, the annual payment is given as 12.3% of the land price. Land price data for each region is given as average price of residential and industrial used spots in the region, reported by the Ministry of Land, Infrastructure and Transportation. DC location cost is given by the required annual payment for the required space discussed above plus fixed cost of setup and maintenance of a DC (5 million yen, annually). As stated before, we consider transportation cost only between DC and retail regions, and ignore that between the plant and DCs. Due to this assumption, we can neglect the effect of plant location. The unit stock cost at the plant h_0 is required to solve the stock allocation problem between plant and DC so that it is set as the average value of h_k s over the 207 regions. (Fixed location cost).

Annual land rent per unit area of plant f_0 has no effect for the optimality of the problem, then we ignore it. The other parameters are set as follows; replenishment interval at plant $\mu_0 = 6$ (days) and that in DCs $\mu_1 = 6$ (days), respectively, stock-out penalties are $\alpha = 600$ (yen) and $\beta = 1200$ (yen). The case with the parameter values described above gives a benchmark case, referred as $B0$.

Other simulation cases are set with variations of demand and supply side parameters. First, cases for different stock-out penalty are set to clarify the influence of demand side needs on DC location and their inventories by holding the ratio of the two stock-out penalty constant, and the other parameters are identical with $B0$. The cases with higher stock-out penalty than $B0$ are indicated by H + numbers, for example, $H1$, and the cases with lower stock-out penalty than $B0$ are indicated by L + numbers, for example, $L1$, respectively. Totally 16 cases for different stock-out penalties are prepared.

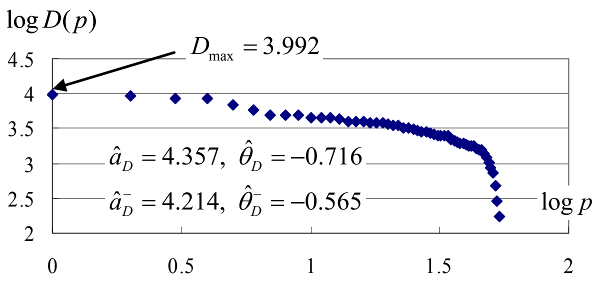

Figure 3 Demand rates and ranks in case $B0$

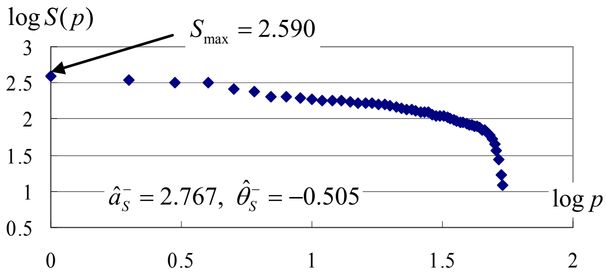

Figure 4 Stock sizes and ranks in case $B0$

Secondly, cases for different replenishment intervals are set to clarify the influence of production and logistic system on DC inventory and location. The cases for shorter replenishment interval are denoted by IS + numbers, while the cases for shorter replenishment interval are denoted by IL + numbers. In this simulation, only the replenishment interval parameter is changed, the other parameters are identical with $B0$. As a result, totally 9 cases reflecting different supply side conditions are set. Thirdly, cases for expressway toll discount are set to clarify the influence of control variable of the government on DC inventory and location, indicated by D + numbers. In this simulation, only the expressway toll included in inter-regional transportation cost parameter is changed, the other parameters are identical with $B0$. Totally 5 different cases are set. The optimization procedure between stock allocation model and facility location model referred in 3.1 is converged about 3 or 4 iterations for all cases; i.e. DC locations are not updated anymore among these models.

Now, we first show the result of the benchmark case $B0$. The logarithm of demand rate at each DC with logarithm of its rank is plotted on Fig. 3. The rank-size coefficient for demand rate $\hat{\theta}_D$ estimated by using whole data is -0.716. However, the rank-size distribution of demand rate has concave shape as Fig.3, the intersection is over-estimated, and $\hat{\theta}_D$ is under-estimated due to the steeper slope around the tail end (lower demand rate areas). Such phenomenon is often reported in the previous studies. In order to avoid this tail end effect on the coefficient estimates, the 10% of tail end data (low ranked data) will be excluded in rank-size coefficient calculation, as following a previous approach (Nitsch, 2005). The rank-size coefficient $\hat{\theta}_D^-$ estimated by the data except 10% of tail end samples is -0.565 to indicate much gradual slope than $\hat{\theta}_D$, so then the influence of tail end data is well alleviated. The logarithm of stock size at each DC with logarithm of its rank is plotted on Fig. 4. Stock size distribution shows rather linear line except tail end. The rank-size coefficient for stock size is -0.505 so that the spatial distribution of stock sizes is more scattered than that of demand rate. It is also more scattered than it expected by Zipf 's law ($\theta = 1$).

4.2 Simulation for different stock-out penalty cases

Tab.1 shows the result of simulated cases with various stock-out penalty. On tab.1, the lowest stock-out penalty case is shown at the second row, then the cases with much higher stock-out penalty are arranged toward the bottom. The location cost shown in tab.1 indicates the summation of f_i s but

Table 1 Optimum DC locations and inventories; for stock-out penalty cases

case	α	β	No. of DC	Total cost	Penalty cost	Transp. cost	Location cost	Safty stock	Ave. s-o.prob.
L6	6	12	59	77936	413	33373	44149	0	45.72%
L5	12	24	59	77949	427	33373	44149	48	44.36%
L4	30	60	59	77973	451	33373	44149	297	31.33%
L3	60	120	59	79292	459	33398	45436	645	15.35%
L2	120	240	56	82705	435	37448	44822	951	4.80%
L1	300	600	54	85155	427	38780	45948	1208	1.56%
B0	600	1200	54	86925	425	39005	47495	1525	0.71%
H1	1200	2400	54	86923	423	39005	47495	1755	0.34%
H2	3000	6000	57	89915	421	37165	52329	1803	0.12%
H3	6000	12000	57	89914	419	37165	52329	1888	0.06%
H4	12000	24000	57	90827	419	37071	53337	2001	0.03%
H5	30000	60000	54	91147	416	39351	51380	2106	0.01%
H6	60000	120000	55	92599	417	38821	53361	2246	0.00%
H7	120000	240000	55	92599	416	38821	53361	2352	0.00%
H8	300000	600000	55	94143	419	39051	54674	2470	0.00%
H9	600000	1200000	55	94147	423	39051	54674	2570	0.00%

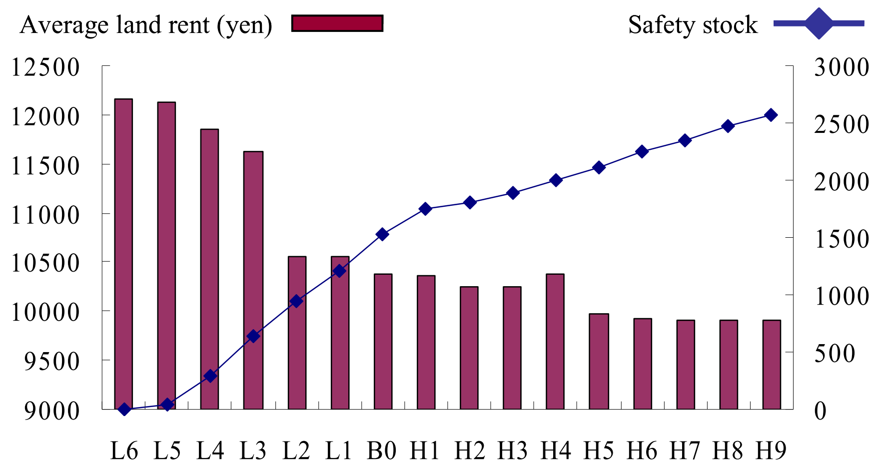

Figure 5 Average land rent and safety stock; for stock-out penalty cases

not include $h_i s_i$ in eq.(17) , for easy understanding the difference of land price. Note that for all these cases, stock at the plant s_0 is almost equal to average demand $\mu_0 \Lambda_0$ as the lowest amount constrained by eq.(8).

For example in $B0$ case, 54 of DCs with totally 1,525 stocks locates and its average stock-out probability (shown at the right-end column) is 0.71%. Number of DCs does not monotonically increase as stock-out penalty parameter increases. Total cost is obtained by summing up with penalty, transportation, and location cost. It increases with almost monotonically, but it is almost constant from $L6$ to $L4$, $B0$ and $H1$, $H2$ and $H3$, $H6$ and $H7$, and $H8$ and $H9$. The share of penalty cost is very low in all the cases, while share of location cost is increased corresponding to the increase in stock-out penalty parameters. Safety stock is monotonically increased in order to avoid high

Table 2 Rank-size coefficient and intersection; for stock-out penalty cases

case	No. of DC	$\hat{a}_D^-$	$\hat{\theta}_D^-$	Maximum demand	$\hat{a}_S^-$	$\hat{\theta}_S^-$	Maximum stock
L6	59	4.192	-0.562	4.064	2.725	-0.565	2.588
L5	59	4.192	-0.562	4.064	2.727	-0.565	2.588
L4	59	4.192	-0.562	4.064	2.728	-0.550	2.588
L3	59	4.225	-0.592	4.064	2.763	-0.561	2.604
L2	56	4.242	-0.597	4.064	2.792	-0.559	2.617
L1	54	4.240	-0.590	4.064	2.784	-0.531	2.628
B0	54	4.214	-0.565	3.992	2.767	-0.505	2.590
H1	54	4.214	-0.565	3.992	2.771	-0.501	2.595
H2	57	4.222	-0.583	3.992	2.791	-0.519	2.601
H3	57	4.222	-0.583	3.992	2.793	-0.515	2.605
H4	57	4.260	-0.618	4.064	2.827	-0.540	2.656
H5	54	4.243	-0.594	4.064	2.806	-0.506	2.661
H6	55	4.245	-0.600	4.064	2.810	-0.507	2.665
H7	55	4.245	-0.600	4.064	2.812	-0.504	2.668
H8	55	4.663	-0.647	4.064	2.815	-0.501	2.673
H9	55	4.412	-0.607	4.064	2.815	-0.501	2.673

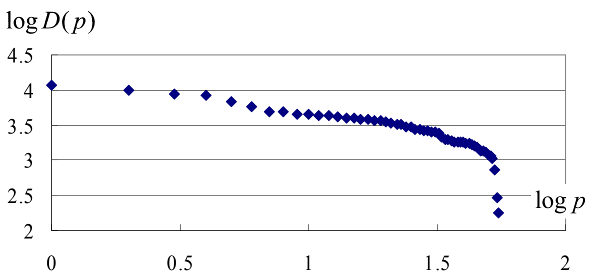

Figure 6 Demand rates and ranks in case $L6$

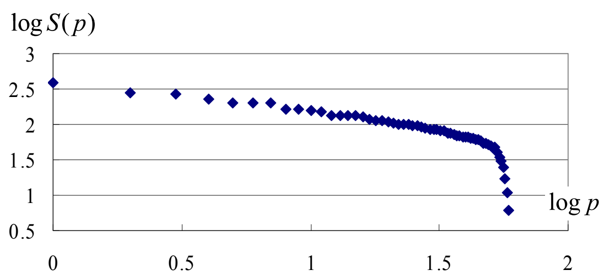

Figure 7 Stock sizes and ranks in case $L6$

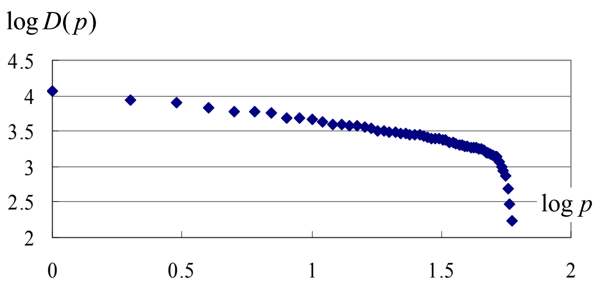

Figure 8 Demand rates and ranks in case $H9$

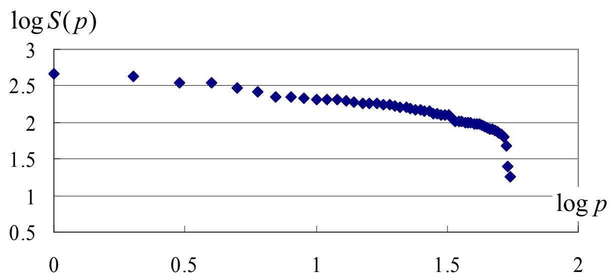

Figure 9 Stock sizes and ranks in case $H9$

average stock-out probability, as the increase in stock-out penalty parameter. Fig.5 shows the average land rent (weighted by stock at each DC) and safety stock. As the stock-out penalty parameter increases, average land rent decreases except at $H4$, therefore, DC location tends to change from higher land rent region to lower land rent region, to realize the sufficient safety stock with moderate location cost. While the slope of safety stock can be approximated by two sections separated at $H1$, the slope of average land rent would be approximated by two (or three) sections separated between $L3$ to $L2$ (and between $H4$ to $H5$). The difference in separation points between the location and

Table 3 Optimum DC locations and inventories; for replenishment interval cases

case	Replenishment Interval (day)	No. of DC	Total cost	Penalty cost	Transp. cost	Location cost	Safety stock	Ave. s-o.prob.
IS4	2	62	72617.67	162.93	30443.76	42011.00	523	6.90%
IS3	3	59	77749.57	226.95	33373.42	44149.24	795	2.88%
IS2	4	58	80878.27	293.90	34455.66	46128.74	1027	1.62%
IS1	5	57	83676.75	357.04	36669.72	46650.01	1220	0.99%
B0	6	54	86924.76	424.58	39004.74	47495.46	1525	0.71%
IL1	7	55	88491.67	491.09	38490.55	49510.08	1555	0.55%
IL2	8	57	90966.70	558.67	37070.76	53337.29	1727	0.45%
IL3	9	55	92806.27	624.00	38820.81	53361.48	1872	0.34%
IL4	10	55	94415.36	690.72	39050.96	54673.69	2018	0.28%

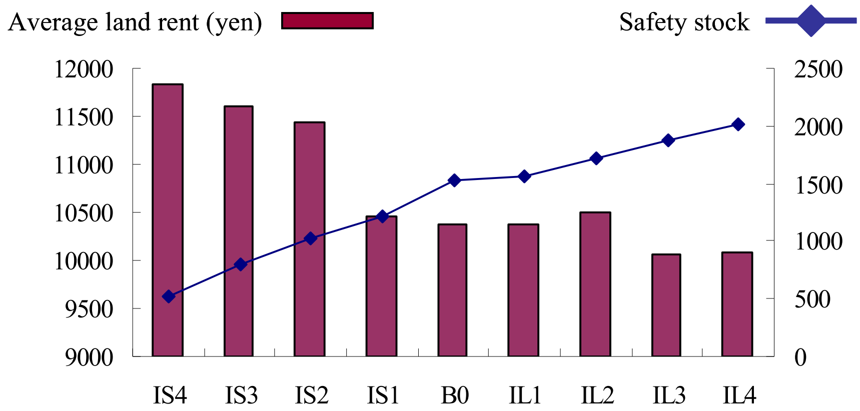

Figure 10 Average land rent and safety stock; for replenishment interval cases

safety stock graphs indicates that drastic change in DC location can be occurred if stock-out penalty from customers increases, even though the required safety stock almost linearly increases from $L5$ and $H1$.

Tab.2 shows the estimates of rank-size coefficient θ and its intersection $a = \log A$ for the same cases. These coefficients are estimated from the data excluding 10% tail-end samples. Rank-size coefficients of demand rate and stock are around -0.5 to -0.6, therefore the distribution of these amounts is more scattered than the Zipf's law. The rank-size coefficient of demand rate $\hat{\theta}_D^-$ becomes slightly smaller, while that of stock $\hat{\theta}_S^-$ becomes larger as the increase of stock-out penalty parameter. The shape of plots are shown in fig. 6, 8, and 3 show a plot of logarithm of demand rate and logarithm of its rank in $L6$, $H9$ and $B0$,respectively. Fig. 7, 9, and 4 show a plot of logarithm of stock and logarithm of its rank in $L6$, , $H9$ and $B0$, respectively. In fig. 6, 8, and 3, shoulder of the distribution in DC demand rate is clearly seen. Note that the estimates for intersection for both $\hat{\alpha}_D^-$ and $\hat{\alpha}_S^-$ are larger than that of maximum value of $S(p)$, which means all the plots have concave shape. As stock-out penalty parameter is set larger, DCs for low demand are increased in number it order to decrease stock-out probability for such areas. In terms of stock distribution, DC with relatively small stock around tail end gathers more stock in fig.9 than in fig.7, because marginal effect to decrease stock-out probability is proportional to the current stock per demand, additional stock to

Table 4 Rank-size coefficients and intersections; for replenishment interval cases

case	No. of DC	$\hat{a}_D^-$	$\hat{\theta}_D^-$	Maximum demand	$\hat{a}_S^-$	$\hat{\theta}_S^-$	Maximum stock
IS4	62	4.158	-0.542	4.063	2.238	-0.478	2.158
IS3	59	4.203	-0.573	4.064	2.453	-0.502	2.336
IS2	58	4.198	-0.566	4.064	2.573	-0.496	2.462
IS1	57	4.256	-0.616	4.064	2.723	-0.544	2.558
B0	54	4.214	-0.565	3.992	2.767	-0.505	2.590
IL1	55	4.218	-0.573	3.992	2.837	-0.514	2.654
IL2	57	4.273	-0.632	4.064	2.939	-0.564	2.757
IL3	55	4.245	-0.600	4.064	2.966	-0.539	2.808
IL4	55	4.244	-0.600	4.064	3.009	-0.539	2.851

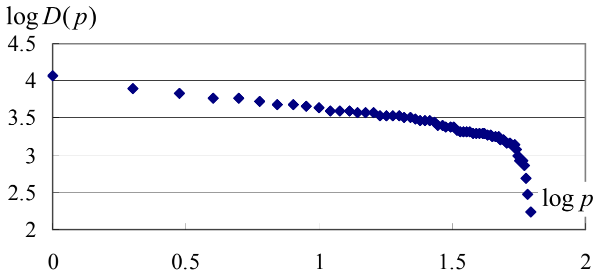

Figure 11 Demand rates and ranks in case $IS4$

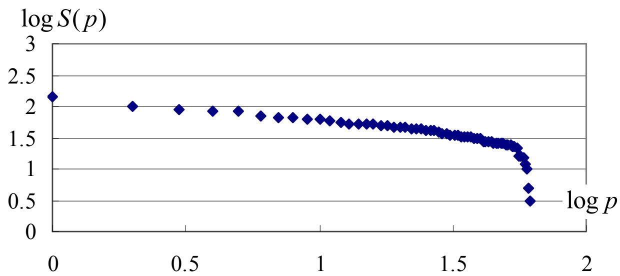

Figure 12 Stock sizes and ranks in case $IS4$

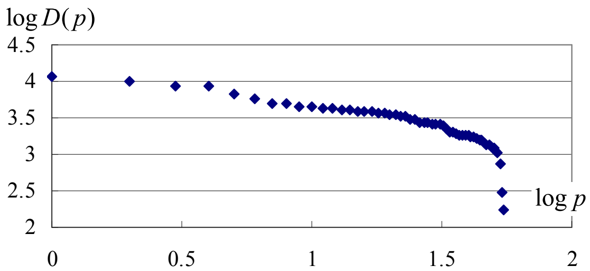

Figure 13 Demand rates and ranks in case $IL4$

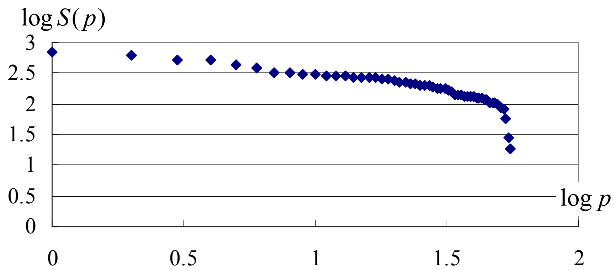

Figure 14 Stock sizes and ranks in case $IL4$

low demand DC can more effectively decrease the total stock-out penalty rather than that to high demand DC.

4.3 Simulation for different replenishment interval cases

Tab.3 shows the result of simulated cases for different replenishment intervals. On tab.3, the shortest replenishment interval case is shown at the second row, then the cases with longer replenishment interval are arranged toward the bottom. Number of DC is decreased from $IS4$ to $B0$, but it does not monotonically change further from $IL1$ to $IL4$, as increase in replenishment interval parameters. Penalty, transportation and location costs are monotonically increase as the increase of replenishment interval parameters. Fig. 10 shows the average land rent (weighted by stock at each DC) and total safety stock. Average land rent is considerably decreased between $IS2$ and $IS1$, while safety stock seems to lineally increase corresponding as replenishment interval parameters increase.

Table 5 Optimum DC locations and inventories; for expressway toll discount cases

case	Discount rate	No. of DC	Total cost	Penalty cost	Transp. cost	Location cost	Safety stock	Ave. s-o.prob.
B0	-	54	86925	425	39005	47495	1525	0.71%
D1	-10%	54	85703	425	37783	47495	1525	0.71%
D2	-20%	57	85124	425	35098	49601	1405	0.73%
D3	-30%	53	82601	424	35918	46259	1366	0.70%
D4	-40%	52	81022	423	35668	44931	1345	0.69%
D5	-50%	50	80541	422	36730	43389	1336	0.66%

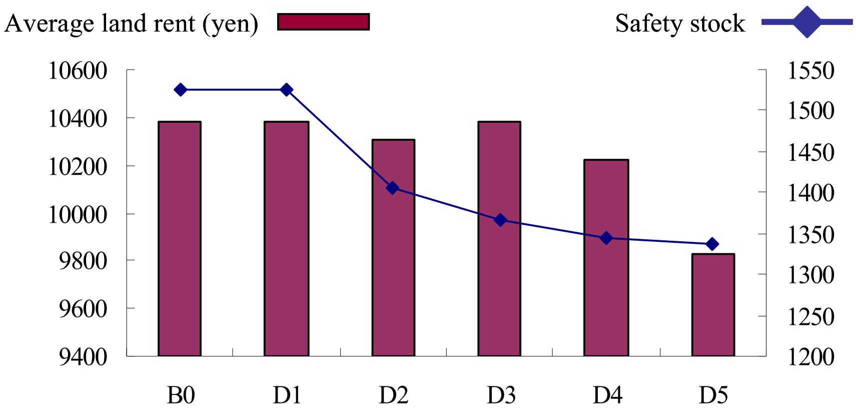

Figure 15 Average land rent and safety stock; for expressway toll discount cases

Since large stocks are required under longer replenishment interval, lower land rent regions become more attractive for DC location.

Tab.4 shows the estimates of rank-size coefficients $\hat{\theta}$ and its intersections $\hat{\alpha}$. Rank-size coefficient of demand rate $\hat{\theta}_D^-$ is around -0.54 to -0.6, slightly becomes smaller, but not monotonically as replenishment interval parameters increase. Similar propensity is seen for the intersection of demand rate $\hat{\alpha}_D^-$. Fig.11 and fig.13 show plots of the demand rate to its rank. Comparing fig.13 and fig.11, demand rate at higher rank DCs is larger in $IL4$ than $IS4$, while that at lower demand rate DCs is larger in $IS4$ than $IL4$. Therefore demand rate becomes more scattered under shorter replenishment interval setting; i.e. quicker production and logistic system. Rank-size coefficient of stock size $\hat{\theta}_S^-$ is around -0.48 to -0.54, which is lower than that of demand rate. Fig.12 and fig.14 show plots of the stock size to its rank. Except upward shift of the stock size distribution from $IS4$ to $IL4$, similar characteristic with the distribution of demand rate can be seen.

4.4 Simulation for different expressway toll discount cases

Tab.5 shows the result of simulated cases for different expressway toll discounting. The second row shows the case $B0$ with no discount, and the other cases from $D1$ to $D5$ are followed in order with discount rate ascending toward the bottom. In case $D1$, only the transportation cost is 10 % decreased comparing to $B0$, the same DC locations and stocks are obtained. Number of DCs is decreased as increase of discount rate, except case $D2$. From case $D3$ to $D5$, location cost is decreased because

Table 6 Rank-size coefficients and intersections; for expressway toll discount cases

case	No. of DC	$\hat{a}_D^-$	$\hat{\theta}_D^-$	Maximum demand	$\hat{a}_S^-$	$\hat{\theta}_S^-$	Maximum stock
B0	54	4.214	-0.565	3.992	2.767	-0.505	2.590
D1	54	4.214	-0.565	3.992	2.767	-0.505	2.590
D2	57	4.232	-0.593	3.992	2.782	-0.529	2.590
D3	53	4.230	-0.578	4.064	2.780	-0.515	2.634
D4	52	4.251	-0.595	4.064	2.800	-0.531	2.634
D5	50	4.260	-0.595	4.003	2.816	-0.538	2.588

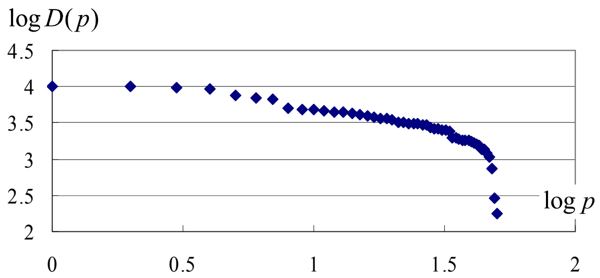

Figure 16 Demand rates and ranks in case $D5$

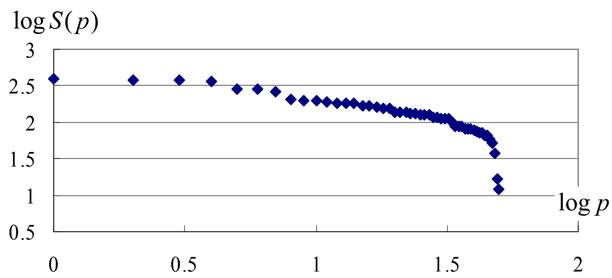

Figure 17 Stock sizes and ranks in case $D5$

a distribution network with fewer DCs and long transportation becomes more efficient due to the discount of expressway toll. Stock-out penalty cost and average stock-out probability are almost constant through all the cases in tab.3. Fig.15 shows the average land rent weighted by stock at each DC and safety stock. Average land rent significantly decrease in case $D5$ along with the decrease of safety stock without increase in stock-out penalty. In our inventory model with stochastic demand, fewer number of DCs with larger inventory is advantageous to alleviate the influence of stochastic demand variation on stock-out penalty cost. Since variance of demand is proportional to squared root of average demand due to the property of Poisson distribution, so then the number of DCs and total safety stock can simultaneously be decreased with constant stock-out probability. Tab.6 shows estimates of rank-size coefficients $\hat{\theta}$ and intersections $\hat{\alpha}$. Rank-size coefficient of demand rate $\hat{\theta}_D^-$ is around -0.55 to -0.6, slightly becomes smaller as discount rate parameter increases. Similar change is also seen in intersection of demand rate $\hat{\alpha}_D^-$. Fig.16 shows a plot of logarithm of the demand rate to its rank. Comparing fig.16 to fig.3, distribution of right side around logarithm of rank being equal to 1.6 to 1.7 ("shoulder" of distribution) in slightly upward in fig.16 than in fig.3, which reflects the decrease of DCs from 54 in $B0$ to 50 in $D5$. Rank-size coefficient of stock size $\hat{\theta}_S^-$ is about -0.5 to -0.54, which is lower than that of demand rate, therefore spatial distribution of stock is more scattered than that of demand rate. Fig.17 shows a plot of logarithm of the stock to its rank in case $D5$. Comparing fig.16 to fig.17, the difference in rank-size coefficients can be confirmed.

5. Summary and conclusion

The location characteristics of distribution centers obtained as the optimal location by the joint inventory / distribution problem is simulated under different sets of parameters reflecting logistics and geographic. The location characteristics of these simulation results are analyzed through the rank-size coefficients estimated from demand rate and stock at DC.

At first, simulation analysis for larger stock-out penalty parameter reflecting the stricter demand side requests clarified that location of DC tends to shift into the area of lower land rent, in order to have large inventory at DCs. Rank-size coefficient becomes lower as the increase in stock-out penalty parameter, suggests that if intolerant customers for stock-out are increased, the needs for DC with large warehouse are also increased at the lower demand area. Secondly, simulation for shorter replenishment intervals shows that if replenishment interval becomes shorter due to efficient production and logistic system, number of DC is increased. Thirdly, simulation for expressway toll discount cases reflecting logistics policy showed that higher discount rate would cause overall change in DC location, along with decrease in safety stock. However, expressway toll discount policy would not much influence on the rank-size coefficients.

Remaining issues are as follows. In this analysis, the statistical problem of estimation of rank-size coefficient is ignored, but it is important to make hypothetical test between two parameters before and after the condition changes. Moreover, the concavity of rank-size distribution should be treated carefully.

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