MULTIACTOR DECISION ANALYSIS FOR REGIONAL INVESTMENT ALLOCATION

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INTRODUCTION

Multicriteria games, which do not necessarily assume a priori aggregation of the criteria or attribute of each player by a utility function, have not been investigated in depth yet. However, this type of repetitive decision situation is of a major interest when preparing a decision support system for realistic problems with many actors involved. An additional dimension of difficulty emerges if some hierarchical aspects are included in the consideration. Hierarchical decision structures have been investigated either in the single criteria fully centralized case, (e.g., Findeisen et al., 1980), or in the fully centralized games of Stackelberg-Germeer type, (Germeer, 1976). Until now, no other reasonable formalization of a decentralized game with hierarchical aspects, allowing for interactive specification of changing multicriteria interests and learning by all actors has been proposed. The purpose of this paper is to explore possibilities of such a formalization, based on a relatively simple but realistic example of regional investment allocation.

Regional investment allocation system includes an upper-level decision maker and lower-level ones interacting and conflicting each other. The upper-level decision maker has preferences on a overall point of view for regional economy and society, and determines the optimal allocation of resources. The lower-level decision makers who do not have the same perception of value and the same information have the financial responsibility for the provision of local public goods. The upper-level decision maker generally does not directly govern the interactions of the lower-level components, but has arbitrated in the dispute between the lower-level decision makers. However, he is not entirely a mediator but has the principal jurisdictional power for influencing the lower-level's behaviors using legistlative, economic and/or other types of policies.

The conflict situation involved in the investment allocation seems difficult to formalize; yet, formalization and abstraction is an important part of the cognition process and thus necessary for a deeper understanding of the multiactor conflict situation. The aim of this paper is not to give a ready solution to our problem, but to present rather general frameworks which allow analyzing the multiactor conflict situation in a satisfactory way. To this end, we attempt to formalize two alternative prototypes of the conflict situation, which may describe the typical features of decision making processes for investment allocation.

First prototype is a monotonous centralized cooperative game. All decisions of players are made by the players jointly acting as a committee; the joint outcomes of these decisions are observed (possibly with help of a model and a decision support system) by the players. Because of the multicriteria nature of the game, the decisions are revised and implemented repetitively, until a satisfactory status quo is attained. Many methods could be adopted to this prototype. In this paper we attempt to develop a multiobjective model with some extended applications of reference point or quasi-satisficing approach (Lewandowski et al., 1982, Wierzbicki, 1985).

Second prototype could be used in attempts to approach the multiactor conflict situation in a more realistic way, by assuming an autonomous game, including the possibility of a non-cooperative behavior of lower-level players: the upper-level decision maker can not directly influence the lower-level but is informed about their interests, and plans this moves to satisficing their goals by observing the lower-level's responses. In this prototype, two variants could be considered. One is a game of Stackelberg-Germeer type, if we assumed that the upper-level player knows precisely the utility functions of the lower-level players. However, we would like to consider that they have an interactive learning situation with changing utilities and thus the classical concept of utility can not be operationalized here. Therefore, we shall consider another variant: the upper-level player has the role of a mediator for other players with multiobjectives, and his prerogations consist only of his larger substantive power, and of his larger information, resulting from his role as a mediator. This mechanism can be also captured by some extensions and elaborations of the solution concepts and procedures adopted in reference point or quasi-satisficing approach as a step-wised mediation process. In the first phase there is a need for mediating procedure to try to lead the lower-level players to a non-cooperative status quo situation that they will both accept. The second phase of interaction is concerned with finding a cooperative solution, starting from the non-cooperative status quo solution.

The issues that can be investigated for such games and would be (a) diverse formalization of the problem, (b) procedures to support obtaining desirable cooperative solutions, (c) principles of decision support for such games, (d) examples of such games, computational and experimental tests. Because of exploratory character of this paper, we do not concentrate on any of the above issues in more detail, trying to cover all of them as a basis for evaluating the reasonability of the general formalization of the problem.

THE FORMULATION OF THE PROBLEM

Because of learning aspects in any decision support system, a model of a game constructed for such purposes must have a repetitive type. However, since the preferences of the players must necessarily change along with learning, we shall not consider here repetitive games explicitly with full description of their dynamics as evolutionary character. We shall present here only one-shot models of the game in one of its repetitions, together with procedures for organizing the repetition and interaction with the decision support system.

Consider three players with decisions $x_i \in X_{0i} \subset R^{n_i}$, $i=1,2,3$. The first player is the upper-level decision maker, but only in the qualitative and procedural sense that : (1) His decision has greater impacts on the overall outcomes of the game than the decisions of other players; (2) His objectives incorporate and aggregate the objectives of other players; (3) He will play also the procedural role of a mediator between other players. The other two players are lower-level decision makers; however, their decisions influence also outcomes of interest to the first player. Thus, we consider here a traditional static and normative model of an autonomous game: $f^{(i)}(x)$, $x \in X_0 \to R^{m_i}$, $i=1,2,3$, are the vectors of outcome mapping of the interests (criteria) of players 1, 2, and 3 respectively, where $X_0 = X_{01} \times X_{02} \times X_{03}$. Without loss of generality, we assume all outcomes $f^{(i)}_j(x)=y^{(i)}_j$, $j=1,---,m_i$,$i=1,2,3$, are to be maximized in the multi-criteria sense. There are, however, additional restrictions on the form of the outcome mapping of the first player. Denote by $E(f^{(1)},X_0)$

the set of cooperative decisions $\overline{x}=(\overline{x}_1,\overline{x}_2,\overline{x}_3)\in X_0$ such that for each $\overline{x}\in E(f^{(1)},X_0)$ the outcome $y^{(1)}= f^{(1)}(x)$ is efficient with respect to the interest of the first player; denote by $E((f^{(2)},f^{(3)}),X_0)$ the set of such cooperative decisions $\overline{x}$ that the pair of outcomes $(f^{(2)}(\overline{x}), f^{(3)}(\overline{x}))$ is efficient with respect to interests of the second and third player's treated jointly. We shall say that the upper-level player shares the interests of the lower-level players, if:

$$E(f^{(1)},X_0)\subset E((f^{(2)},f^{(3)}),X_0), \quad f^{(1)}(x)= \psi(f^{(2)}(x),f^{(3)}(x)). \tag{1}$$

If the function ψ is a transformation of $f^{(2)}$ and $f^{(3)}$, then a sufficient condition for the relation (1) is that ψ is strongly monotone in $f^{(2)}$ and $f^{(3)}$, i.e., outcomes of interests to the upper-level player increase with the outcomes of interests to the lower-level players. However, the requirement of monotonicity of ψ might be too strong. For example, the upper-level decision maker could be interested not only in the well-being of both lower-level decision makers, but also in a balance of their well beings; thus, ψ might depend positively both on the sums of components $f^{(2)}$ and $f^{(3)}$ as on the differences between them. The issues of less stringent general requirements on the form of the function ψ that would be sufficient for the property of sharing interests (1) is open for further investigation; here we assume only that (1) holds and that the lower-level players are aware of this fact, hence they might accept the upper-level player as a mediator.

A MONOTONOUS CENTRALIZED COOPERATIVE GAME

The mathematical properties of cooperative solutions in the game of type I are known; many properties of various cooperative solutions to games with the single interest of each player have been extended to multicriteria case by Krus and Bronisz (1985). An interactive learning process based on a model of the game and a decision support system can be organized as follows (Wierzbicki, 1984,1985):

(1)Step 1: Players use the model of the game in a simulation mode while making decisions without system support; after several repetitions, the outcomes of the last repetition $\tilde{y}_t=(\tilde{y}^{(1)}_t,\tilde{y}^{(2)}_t,\tilde{y}^{(3)}_t)$ are taken as a current status quo point;

(2)Step 2: The decision support system solves $m_1 + m_2 + m_3$ static maximization problems

$$y^{(i)}_{j,max}= \max_{x\in X_0\cap X_i(\tilde{y}_t)} f^{(i)}_j(x) \quad , j=1,---,m_i \ ,i=1,2,3 \tag{2}$$

where $X_i(\tilde{y}_t) = \{x \in R^{n_1+n_2+n_3}: f^{(i)}(x) \geq \tilde{y}_t^{(i)}\}$, i=1,2,3, and informs the players about their maximal scales of achievement,

$$\Delta\tilde{y}_j^{(i)} = y_{j,max}^{(i)} - \tilde{y}_{j,t}^{(i)} \;, \; j=1,---,m_i \;, i=1,2,3. \qquad (3)$$

(3)Step 3: The players are asked to specify their aspirations $\bar{y}_j^{(i)}$ such that $\tilde{y}_{j,t}^{(i)} < \bar{y}_j^{(i)} < y_{j,max}^{(i)}$, j=1,---, m_i, i=1,2,3. These aspirations determine the weight

$$\lambda_j^{(i)} = (1/(\bar{y}_j^{(i)} - \tilde{y}_{j,t}^{(i)}))/\sum_k^{m_i}(1/(\bar{y}_k^{(i)} - \tilde{y}_{k,t}^{(i)})) \; j=1,---,m_i, i=1,2,3. \qquad (4)$$

(4)Step 4: The decision support system computes an efficient cooperative joint decision $\hat{x}$ and its outcome $\hat{y} = (f^{(1)}(\hat{x}), f^{(2)}(\hat{x}), f^{(3)}(\hat{x}))$ by solving the following scalar maximization problem:

$$\max_{x \in X_0} (\min_i \min_{1 \leq j \leq m_i} \lambda_j^{(i)}(y_j^{(i)} - \bar{y}_j^{(i)}) + \varepsilon \sum_i^3 \sum_k^{m_i} \lambda_k^{(i)}(y_k^{(i)} - \bar{y}_k^{(i)})) \qquad (5)$$

where, $y_j^{(i)} = f_j^{(i)}(x)$, j=1,---,m_i, i=1,2,3 and $\varepsilon > 0$ is a small number introduced to eliminate degenerate cases with weakly efficient solutions. Let be the solution of (5) $\hat{y}(\tilde{y}_t)$.

(5)Step 5: This cooperative solution and outcome is presented to the players; if the players jointly accept it, the procedure stops. If they do not accept it, the decision support system modifies the current status-quo point from y to the (attractive) point $\tilde{y}_{t+1} = \tilde{y}_t + \alpha_t (\hat{y}(\tilde{y}_t) - \tilde{y}_t)$ with $0 < \alpha_t < 1$, $\sum_t^{\infty} \alpha_t = \infty$, $\sum_t^{\infty} (\alpha_t)^2 < \infty$ - as an example we can take $\alpha_t = (1/2)^t$, where t is the number of repetitions of the game at this stage, and returns to step 2 with this new status quo point. Such a process, which is a generalization of the Raiffa-Kalai-Smorodinsky cooperative solution concept, (Kalai and Smorodinsky, 1975), gives to the decision makers opportunity to learn and adapt their aspirations. While assuming a non-stationary but convergent utility functions of all players, questions of detailed properties of limit solutions (which are obviously efficient) of such a process might be investigated in the future.

AN AUTONOMOUS GAME WITH A NON-COOPERATIVE BIHAVIOR

The properties of the game of the type II are much more complicated. After a decision x_1 of the player 1, the players 2 and 3 can choose decisions that either lead to a disequilibrium point, or to a non-cooperative equilibrium point that is not necessarily efficient or to a cooperative and efficient solution point.

A multicriteria non-cooperative equilibrium point $\tilde{x}=(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)$ is defined as such a point $\tilde{x} \in X_0 = X_{01} \times X_{02} \times X_{03}$ that, given $\tilde{x}_k$ for $k \neq i$, $\tilde{x}_i$ is an efficient solution for the i-th player. Thus, for instance, $\tilde{y}^{(1)} = f^{(1)}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)$ is an efficient outcome for the player 1 with respect to his decision $\tilde{x}_1$ while $\tilde{x}_2, \tilde{x}_3$ are fixed. The basic difficulty here is that multicriteria non-cooperative equilibria are typically non-unique and thus there is no guarantee that a multi-decision $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)$ composed of these independently selected decisions, - even if each of them was intended to correspond to a non-cooperative equilibrium, - will indeed correspond to a non-cooperative equilibrium. This way, disequilibrium solutions might be selected and even processes of conflict escalation start, (Wierzbicki, 1984). One of the issues of future investigation might be the question, under which conditions such process of conflict escalation can occur. Here we shall simply assume that a current status quo point, obtained for example through several rounds of simulation gaming, is not necessary a non-cooperative equilibrium and might even represent results of conflict escalation.

Multicriteria non-cooperative equilibria are not necessarily efficient solutions in the space of all outcomes jointly considered, which is a known fact even for games with single interests. Thus, there is a question of a mediation procedure while the role of a mediator can be assumed by the first player, that would lead the game to cooperative, jointly efficient solutions. We propose here the following procedure, similar to the procedure for the type I of decision situation.

(1)Step 1: Players use the model for several rounds in the simulation mode and establish thus a current status quo point $\tilde{y}_t$.

(2)Step 2: A decision support system computes maximal scales of achievement $\Delta \tilde{y}^{(i)}_j = y^{(i)}_{j,max} - \tilde{y}^{(i)}_{j,t}$ by solving (2) and informs all players about them.

(3)Step 3: The players are asked to specify their aspirations $\bar{y}^{(i)}_j$ such that $\tilde{y}^{(i)}_{j,t} < \bar{y}^{(i)}_j < y^{(i)}_{j,max}$, whereby players 2,3 are supposed to consult their aspirations with player 1 but are free to set them finally autonomously. While informing about them the player 1 these aspirations determine the weighting coefficients (4).

(4)Step 4: The system computes an efficient cooperative decision $\hat{x}$ and its outcome $\hat{y}$ by solving (5). These results, however, are communicated only to the player 1. To all players, the system communicates only an improved point $\tilde{y}_{t+1} = \tilde{y}_t + \alpha_t(\hat{y}(\tilde{y}_t) - \tilde{y}_t)$ with α_t , say, of the form $\alpha_t = (1/2)^t$, together with the corresponding decision $\tilde{x}_{t+1}$ such that $\tilde{y}_{t+1} = (f^{(1)}(\tilde{x}_{t+1}), f^{(2)}(\tilde{x}_{t+1}), f^{(3)}(\tilde{x}_{t+1}))$, computed additionally by the system for this purpose.

(5)Step 5: The players 2 and 3 are asked to consult their actual decisions in this repetition of the game with player 1. He announces and executes his decision $\tilde{\tilde{x}}_1$ first, then the player 2 and 3 follow with decision $\tilde{\tilde{x}}_2$, $\tilde{\tilde{x}}_3$. The model simulates the outcomes of the game $\tilde{\tilde{y}} = (f^{(1)}(\tilde{\tilde{x}}), f^{(2)}(\tilde{\tilde{x}}), f^{(3)}(\tilde{\tilde{x}}))$ with $\tilde{\tilde{x}}=(\tilde{\tilde{x}}_1, \tilde{\tilde{x}}_2, \tilde{\tilde{x}}_3)$. This outcome is taken as next status quo point upon returning to step 2. Observe that the players are free to choose decisions $\tilde{\tilde{x}}_1$,$\tilde{\tilde{x}}_2$,$\tilde{\tilde{x}}_3$ different than $\tilde{x}_{1,t+1}$, $\tilde{x}_{2,t+1}\tilde{x}_{3,t+1}$ and thus there is no guarantee that this procedure will converge to a cooperative, jointly efficient solution. On the other hand, the influence of the first player acting as the mediator can result in such convergence; although he cannot force other players to follow decisions suggested by the system, he can change his decisions in the next round unfavorably for any players that does not follow his advice. Since we assumed that he shares the interests of other players, he is motivated to bearing the outcomes to the joint efficient frontier. The proposed procedures of interactive decision support give rise to many theoretical question which will not be addressed here in more detail became of exploratory character of this paper.

REGIONAL ALLOCATION OF INVESTMENT

Let us consider here a simplified but realistic example of a hierarchical multicriteria game dealing with regional investment allocation. A hierarchical structure with the upper-level player, prefectural government and two lower-level player, local governments, are considered. Let us assume each player has only one decision variable: player 1 decides the subdivision of resources between players 2 and 3. x_1 represents the portions of resources allocated to player 2. Both player 2 and 3 allocate their resources into two different sectors, industry and housing sectors. x_i for i=2,3 mean the portions of resources allocated in the housing sector. All decision variables $(x_1, x_2, x_3) \in R^3$ are constrained by $0 \leq x_i \leq 1$, for i=1,2,3. Let us assume that player 2 and 3 intend to maximizes two criteria, the number of inhabitants and of employments in their jurisdictions, i.e., $f_1^{(i)}(x)$ and $f_2^{(i)}(x)$, i=2,3. Player 1 is assumed to maximize two criteria, i.e., the number of inhabitants and of employments in the whole region, which are the sums of the attainment levels of criteria of player 2 and 3. A regional econometric model (Yoshikawa et al., 1986) could provide the payoff values of multicriteria. Being upon the outcomes of the econometric model, the payoff functions, $f_j^{(i)}(x)$, i=1,2,3, j=1,2 are approximated to take quadratic forms, Table. 1 Solution of the game I

		$\tilde{y}_t$	y_{max}	$\bar{y}$	$\hat{y}(\tilde{y}_t)$	$\tilde{y}_{t+1}$
P1	1.Inhabitants	1318588	1332430	1325513	1327476	1323049
	2.Employments	620714	698188	659451	698188	659451
P2	1.Inhabitants	423296	456005	440012	442106	433063
	2.Employments	168108	275414	221761	275414	221761
P3	1.Inhabitants	895292	920366	907468	885372	889986
	2.Employments	452606	482434	467521	422774	437690
decision variables (x_1, x_2, x_3)		Point A (0.5,0,0)			Point B (1,0,0)	(0.75,0,0)

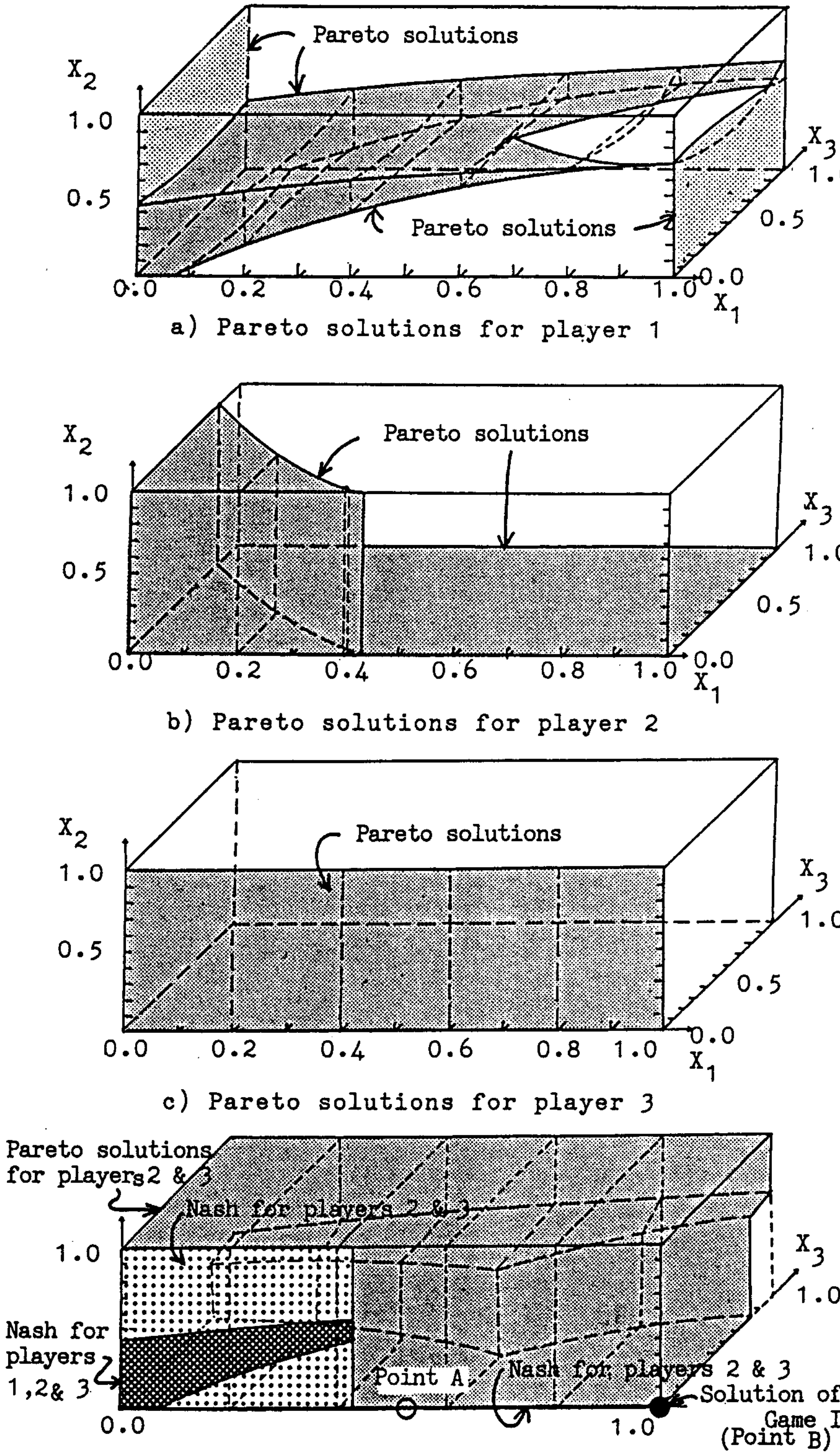
$$f_j^{(i)}(x) = \alpha^{ij} + \sum_k^3 \beta_k^{ij}(x_k - 0.5) + \sum_k^3 \sum_\ell^3 \gamma_{k\ell}^{ij}(x_k - 0.5)(x_\ell - 0.5) \tag{6}$$

where α, β, γ are coefficients; e.g. taking the following values for criterion $f_1^{(2)}$, $\alpha^{21}=430623$, $\beta_1^{21}=22281$, $\beta_2^{21}=-8874$, $\beta_3^{21}=7960$, $\gamma_{11}^{21}=4952$, $\gamma_{12}^{21}=-23078$, $\gamma_{13}^{21}=-1755$, $\gamma_{22}^{21}=5411$, $\gamma_{23}^{21}=-9534$, $\gamma_{33}^{21}=-11910$. Figs.1-a,-b,-c explain Pareto solutions for player i, i=1,2,3, with respect to his two criteria given the solutions of the other players, respectively. Fig. 1-d illustrates multiobjective Nash non-cooperative solutions between players 2 and 3, and those between players 1,2, and 3. Pareto cooperative solutions by players 2 and 3, (which are also Pareto for player 1 in our case) are also shown in Fig. 1-d.

Attempts to reach agreement on multi-lateral action always start with an assessment of the noncooperative status-quo point. Let us assume that the status quo point is given as the point A in Fig. 1-d. The status-quo point A can be characterized by that player 1, upper-level player is assumed to be neutral in subdivision of his resources to players 2 and 3, and players 2 and 3 intend to correspond to a noncooperative equilibrium. Table 1 summarizes the first-round outcomes of our experiences on game of type I , given the reference points of players as shown in Table 1. The position of the compromised solution of games I in the decision space is illustrated in Fig. 1-d.

CONCLUSION

This paper has dealt with a hierarchical multiactor decision analysis with a realistic example of regional investment allocation in Japan. A hierarchical structure with one actor in the upper-level and



a) Pareto solutions for player 1

b) Pareto solutions for player 2

c) Pareto solutions for player 3

d) Multiobjective Nash and Pareto solutions

Fig.1 Multiobjective Pareto Solutions in Multi-actor Decision Problem.

two actors in the lower-level are considered. Each actor is assumed to have his own multiple criteria and so the problem forms a multicrteria game. Two prototypes of the game have been considered. One is a cooperative game and the other is an autonomous game. Procedures of interactive decision support are proposed for both cases. Such computational and empirical tests have been made by using a simple model of the regional investment problem in a prefecture in Japan.

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