# **Modeling the Moped Flow Aiming Discrete Time Targets**

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**Abstract:** There are land uses that are substantial trip attractors during the morning commuting peak hour. However universities are unique in the existence of several additional traffic peaks during a day. A model for students' moped flow based on parking congestion cost and arrival (departure) time is proposed and validated. The model is theoretically formulated considering the equilibrium conditions and numerically validated using a bidirectional, real dataset collected and classified on the road connecting the Kawauchi and Aobayama Campuses of Tohoku University, Sendai, Japan. This model captures the effect of congestion such as the presented on the parking lots; and also, the effect of the queue in terms of psychological discomfort.

Key Words: departure time, commuter behavior, equilibrium conditions

#### 1. INTRODUCTION

## 1.1 Background

Commuter behavior modeling is an important topic in different contexts. Human geographers are interested in mobility, transport economy and management, traffic engineers are interested in traffic flow theories, while road engineers focus on traffic accident research and prevention; where the main goal is user-oriented and practical in nature. In this sense, it is important to define mathematical models based on economic behavioral assumptions, and also to validate them

Various proposed strategies to combat peak-period problems are based on the idea of diffusing the concentration of peak period demands (Henderson, 1974 and Henderson, 1981). Strategies of this type include: flexible time work policies (Yoshimura an Okumura, 2001) and congestion pricing (Arnott, R *et al.*,1990). Their intention is to encourage commuters to alter the departure time of travel to work. If some commuters adjust their departure time, then the commuter travel can be more uniform across the commute period, thereby reducing the peaking effect. However, when the traffic congestion has more than one peak during a day, solutions to one specific peak time, may result into shifting congestion to other commute period(s). In this sense, commuting to a university is a direct example of how several congestion peaks are presented in a day.

Student commuters using a moped clearly reflect the preferred arrival times near the starting times of classes, since they are less restricted by road capacity. Still, they also come across the congestion problem at the parking lot in the most preferable minutes. Student commuters, therefore, may decide their arrival time in the trade off between convenient schedule and congestion troublesome. This research describes the students' moped flow when they commute to classes. Such description and its validations are the starting point to forecast the flow and also how the student's behavior will be following any policy interventions, such as,

changes in the class schedules, share shift of attending students of classes, reduction in the number parking lots or traffic control measures such as signalization.

#### 1.2 Literature Overview

The commuters' decision of departure time is of fundamental importance to the study of peak-period traffic congestion and to the analysis of traffic control as well as broader demand-side congestion relief measures, such as pricing and ride sharing incentives. When a trip occurs during the peak time hours, the commuter incur in waiting time due to congestion. We need a balanced approach over the collective process of congestion and individual reaction behavior to it. Especially in case of students commuting, whose value of time is considered to be more concentrated than general travelers. For that reason, a macroscopic model becomes an efficient approach.

Early research works deal with the cost associated to congestion (Henderson, 1974 and Henderson, 1981). While other studies relate the social optimum equilibrium which occur during the congestion in a bottleneck (Arnott *et al.*,1990). Over the past decades, there have been active researches on the departure time problem, both in econometric modeling and dynamic user equilibrium (DUE) analysis fields. Detailed discussions can be found in De Palma *et al.*, 1983. Daily departure time choice (Jou *et al.*, 2008) is discussed by means of prospect theory, which relies on: the earliest acceptable arrival time and the work starting time for a given commuter. The results obtained from this research said that preferred arrival times of commuters tend to be near their work starting times. Additionally, Shannon *et al.* (2005) found for university commuters on both staff members and students; that the most significant barrier for active commuting is travel time.

The model developed in this research describes the departure time choice while there are multiple time targets. Also, a macroscopic approach of the traffic flow is used. Since daily traffic and daily capacity does not have a significant gap, but still presents congestion during peak hours, then a temporal distribution of the commuter traffic is analyzed.

#### 2. MODEL FORMULATION

### 2.1 Model Approach and Notation Used

This research focus on explaining the arrival (departure) time decision choice of student commuters with mopeds as their mode choice of traveling. Based on the macroscopic traffic flow models proposed by Arnott *et al.* (1990) and Yoshimura and Okumura (2001); a time-based disutility function is formulated; and, their behavior considering their arrival (departure) time choice is measured in terms of (dis)utility related to the parking congestion and schedule cost. Therefore, we consider that the student commuter using a moped as his travel method can choose more freely than working commuters; therefore, the congestion that the student commuter must face is with other students.

In the formulations we do not consider the differences in ordinal commuting time (in non-congested situations) over different housing location, because such difference is shifted with the housing cost differences in the housing market in longer-run. Consequently, we only consider the additional travel time due to congestions in our models.

The following notation is used in the models:

U(t) is the disutility level of arrivals at time t.

 $\bar{U}$  is the disutility level of arrivals at equilibrium.

V(t) is the disutility level of departures at time t.

 $\overline{V}$  is the disutility level of departures at equilibrium.

x is the moped vehicle queue in the parking lot.

 $\frac{dx}{dt}$  is the moped vehicle flow.

et is the early cost of a student who departs from home at time t. e < 0 because early t gives a larger cost.

lt is the late cost of a student who departs from home at time t. l < 0 because early t gives a larger cost.

 $T_k$  is the target time of the morning class k at time T.

 $(t + \kappa \dot{x})$  arrival time to classroom at time t and delay from moped flow  $\dot{x}$ .

 $\lambda x$  is the psychological trouble of walking from the parking lot to the classroom given the queue of mopeds x in the parking lot at time t.

 $(t - \delta \dot{x})$  arrival time at home, when leaving campus at time t and delay from moped flow  $\dot{x}$ .

 $[t-TF_k]$  is the waiting time at campus before the departure at time t after  $TF_k$  as the Finish Time of the class k.

 $\psi x$  is the psychological trouble of walking from the classroom and pass through the parking lot given the queue of mopeds x in the parking lot at time t.

 $e, c, \kappa_e, \lambda_e$  are parameters associated to the early arrivals.

 $l, d, \kappa_l, \lambda_l$  are parameters associated to the late arrivals.

 $\eta, \delta, \iota, \psi$  are parameters associated to the late departures.

### 2.2 Arrival Model

Let the disutility of the student who leaves home at time t and arrives the classroom before the start time of the k th class at  $T_k$  be defined as:

$$U(t) = et + c[T_{k} - (t + \kappa_{c}\dot{x})] + \lambda_{c}x \tag{1}$$

The encountered moped flow by other students trying to reach the same k class makes a delay in the arrival time  $(t + \kappa_e \dot{x})$ ,  $[T_k - (t + \kappa_e \dot{x})] > 0$  then defines the waiting time on campus after the before the start time  $T_k$  of the class k. The terms that compose equation (1), relate the cost of leaving home early, the cost of waiting on campus and the discomfort of parking far from the classroom.

The equilibrium condition says that no student can decrease his/her disutility by changing the arrival (departure) time. In this sense, the disutility is irrelevant to the departure time t, therefore:  $U(t) = \overline{U}$ , if  $\dot{x} > 0$  and  $U(t) > \overline{U}$ , if  $\dot{x} = 0$ . In other words, this formulation explains that the student commuter using a moped pays certain cost from leaving his house early (in terms of disutilty) and must wait before his class start, but is compensated with less congestion encountered and a more comfortable parking, compared with the commuter who tries to arrive close to  $T_k$  for the same k-th class.

The differential equation for the equilibrium condition is solved, and the following result for

the queue x are obtained (with  $IC_{UE}$  as the integration constant):

$$x = IC_{UE}e^{\frac{\lambda_e}{\kappa_e c}t} + \left[ (T_k - \frac{U}{c})\frac{c}{\lambda_e} + \frac{(1 - \frac{e}{c})}{\kappa_e} \frac{(\frac{\lambda_e}{\kappa_e c}t - 1)}{(\frac{\lambda_e}{\kappa_e c})^2} \right]$$
 (2)

With the following derivative describing the flow of mopeds:

$$\dot{x} = \frac{dx}{dt} = IC_{UE}e^{\frac{\lambda_e}{\kappa_e c}t} \left(\frac{\lambda_e}{\kappa_e c}\right) + \frac{\left(1 - \frac{e}{c}\right)}{\frac{\lambda_e}{c}}$$
(3)

In the same way, a student who arrives at  $(t + \kappa_l x)$  after the class start time  $T_k$ , let the disutility be defined as:

$$U(t) = lt + d[(t + \kappa_l \dot{x}) - T_k] + \lambda_l x \tag{4}$$

where,  $d[(t + \kappa_l \dot{x}) - T_k] > 0$  defines the cost of delay of arrival produced by the flow of mopeds  $\dot{x}$  at time t.

Again with  $IC_{UL}$  as the integration constant, the solutions are:

$$x = IC_{UL}e^{\frac{\lambda_l}{\kappa_l d}t} + \left[\frac{\left(\frac{l}{d}+1\right)}{\left(\kappa_l\left(\frac{\lambda_l}{\kappa_l d}\right)^2\right)}\left(\frac{l}{\kappa_l d}t+1\right) - \left(\frac{\overline{U}}{d}+T_k\right)\frac{d}{\lambda_l}\right]$$
(5)

$$\dot{x} = \frac{dx}{dt} = IC_{UL}e^{\frac{\lambda_l}{\kappa_l d}t} \left(\frac{\lambda_l}{\kappa_l d}\right) + \frac{\left(\frac{l}{d} + 1\right)}{\frac{\lambda_l}{d}}$$
(6)

At this point the Arrival Model, in case of the early comer, describes the student that select its arrival time to minimize the disutility in terms of cost of leaving his house early, the waiting time in the classroom before his class begins and parking troublesome; in contrast, the late comer, faces less cost of leaving his house early, the time of class he missed because of his tardiness and the trouble of parking far. Also, this disutility function accounts the congestion associated to moped flow in both cases, arriving early or late; the student commuter is aware to some extent of the delay produced from the moped flow congestion when selecting his arrival time.

The time  $t^*$  is defined as the commute start time of the student who just reaches the classroom at time  $T_k$ , in other words:

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$$T_{k} = t^* + \kappa_{l} x(t^*) \tag{7}$$

When equation (7) is substituted on equation (1) then at time  $t^*$ , with  $\lambda_e$  as the parameter  $\lambda$  associated to the disutility for early arrivals is defined as:

$$U(t^*) = et^* + \lambda_{\mathcal{L}}x(t^*) = \overline{U}$$
(8)

In the same way, with  $\lambda_1$  as the parameter  $\lambda$  associated to the disutility for late arrivals:

$$U(t^*) = lt^* + \lambda_{,x}(t^*) = \overline{U}$$

$$\tag{9}$$

It is possible to obtain all the parameters  $e, c, l, d, \kappa_e, \lambda_e, \kappa_l, \lambda_l$  as well as the integration constant  $IC_{UE}$ ,  $IC_{UL}$  and the disutility  $\overline{U}$  endogenously with the bidirectional real dataset collected.

Previous works dealing with the working commuter (Yoshimura and Okumura, 2001) have an arrival time fixed. This formulation allows the late arrivals, since the moped late arrivals for the k-th class and the early arrivals for the k+1 th class compose the traffic composition.

## 2.3 Departure Model

Let the disutility of the student who leaves the classroom at time t after the k th class finishes at time  $TF_k$ , be defined as:

$$V(t) = \eta(t - \delta \ddot{x}) + t[t - TF_k] + \psi x \tag{10}$$

Because the exiting flow is defined as  $-\dot{x}(t) \ge 0$  by the decrease of parking mopeds, the delay is given by  $-\delta \ddot{x}$ . The terms that compose equation (10) relate the cost of late arrival at home, waiting cost before departure from campus and the discomfort of passing the filled parking lot.

The equilibrium is described as:  $V(t) = \overline{V}$ , if  $\dot{x}(t) < 0$  and  $V(t) > \overline{V}$ , if  $\dot{x}(t) = 0$ . Following the procedure similar to the Arrival Model, we solve the differential equation to obtain the student commuter departure time. Then the following equation describes the queue of moped with  $IC_V$  as the integration constant:

$$x = IC_{v}e^{\frac{\psi}{\eta\delta}t} + \left[\frac{1}{\psi}(tTF_{k} + \overline{V}) + \frac{(\eta + \iota)}{\eta\delta}\left(\frac{1 - \frac{\psi}{\eta\delta}t}{\left(\frac{\psi}{\eta\delta}\right)^{2}}\right)\right]$$
(11)

The next derivative describes the exiting flow of mopeds:

$$\dot{x} = \frac{dx}{dt} = IC_V e^{\frac{\psi}{\eta \delta}t} (\frac{\psi}{\eta \delta}) - \frac{\eta + \iota}{\psi}$$
(12)

#### 3. DATA

The dataset to estimate the model parameters consists of the traffic measured by a supersonic traffic counter instrument. This dataset corresponds to the counting of Aobabashi site which is located between the Kawauchi Campus and Aobayama Campus of Tohoku University, for arrival (westward) traffic and exiting (eastward) traffic. A sample of 25 days was taken during December 2007 when classes were in session. The detailed location is shown in Figure 1.

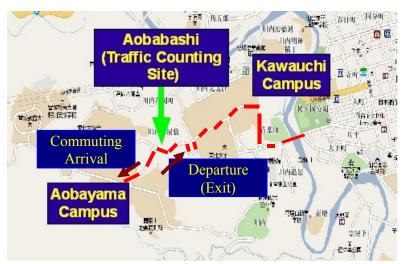


Figure 1 View of Tohoku University campuses

The data are composed of three types of vehicles (Small, Medium and Large). The proposed length for the moped (as a small vehicle) in this study is up to 3 meters long. When the commuting data were classified, there are visible peaks which are known to occur close to the beginning time of the classes held by the university. Since the study focus on mopeds, then the sample of the small vehicle type is used. On Figure 2 there are visible peaks associated to the class timetable, especially in the small vehicle flow. In order to determine the parameter values from the traffic flow samples, the following considerations where used:

- The moped traffic flow is low in the early morning, and starts to increase around 7:30 hrs, this time is selected as the starting time  $(t_0)$  for the early arrivals for the first class  $(T_1)$  at 8:50 hrs.
- Upon reaching the start time of the first class  $(T_1)$ , some late arrivals start, and also early arrivals for the next class  $(T_2)$  at 10:30 hrs. Therefore a mixture of the two flows where considered to model this portion of the time range.
- In the next interval between the second  $(T_2)$  and the third class  $(T_3)$ , the early arrivals start at usual but the finish time for the second class  $(TF_2)$  is 12:00 hrs. Therefore, calculations for the second class  $(T_2)$  late arrivals finish at noon.
- The last class  $(TF_5)$  finish at 17:50 hrs. This hour marks the finish time for the late arrivals associated to this class.

According to the traffic composition segregation and the 5-minute counting, moped flow is erratic and uneven before 7:30. At this time, the moped flow increases steadily, and reach its first peak near 8:50 hrs, which is the starting time of the first morning class at Aobayama

Campus. Based on the model formulation and assumptions, the university's class timetable and the traffic composition segregation, we determine that the timeline for the following analysis starts at 7:30 hrs.

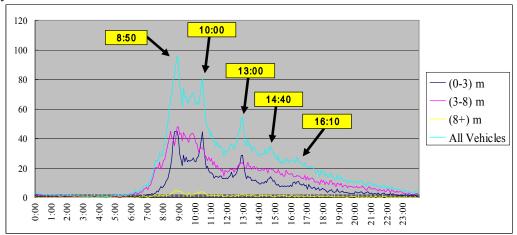


Figure 2 Counted arrival traffic patterns classified by vehicle size type

Regular traffic and the student's behavior occur during regular days, so the sample does not contain the days when the traffic is lesser, such as the weekends. Also, the number of classes changes from day to day in a week. Then, the sample was divided into five weekdays (Monday through Friday).

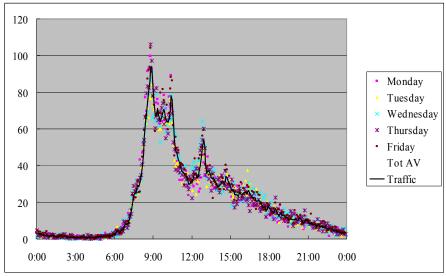


Figure 3 Counted arrival traffic patterns classified by weekday

Finally, the time segregation of the sample was done separating the traffic counting using 5-minute intervals as seen Figure 3 and the total traffic count by day and by moped (small vehicle) is presented in Table 1. Consistently with Jou *et al.* (2008), the student commuter, behaves as the working commuter cited by their work, in other words he tends to arrive closely to the class starting time. However, the student commuter does not arrive all weekdays at the same time; according to his schedule, various target times occur during the week; this type of behavior and commuting schedule differs from the working commuter.

Table 1 Daily arrival traffic counts in December 2007

	Sunday	Monday	Tuesday	Wednesda	Thursday	Friday	Saturday
Day							1
Total Count							3070
Small Vehicle Count							944
Day	2	3	4	5	6	7	8
Total Count	2497	5957	5736	5861	6084	6054	3069
Small Vehicle Count	679	2209	2019	2034	2247	2434	878
Day	9	10	11	12	13	14	15
Total Count	2105	5960	5864	5990	5491	6176	3271
Small Vehicle Count	712	2319	2110	2254	2718	2608	940
Day	16	17	18	19	20	21	22
Total Count	2148	5540	5960	6009	6014	6201	3128
Small Vehicle Count	704	2036	2121	2199	2213	2323	855
Day	23	24	25				
Total Count	1843	2378	5129				
Small Vehicle Count	720	690	1377				
Day Average	Sunday	Monday	Tuesday	Wednesda	Thursday	Friday	Saturday
Total Daily Average	2148 25	5819	5672 25	5953 333	5863	6143 667	31345
Smal%	32 8%	37.6%	33 6%	36.3%	40.8%	40.0%	28.8%

#### **4 ESTIMATION RESULTS**

#### 4.1 Parameter Estimation for the Arrival Models.

The parameters were estimated using minimization of the sum of square errors for each period considered in the time range previously described. Since the estimation results are nonlinear minimization in nature, the existence of the unique solution was not theoretically proven. In order to obtain the global minimum, several points were used as a starting value of the minimization process. In case of a local minimum the values were compared until no other minimum point was obtained. In this sense, it can be said that the global minimum was obtained, and all conditions were met. Also, the unit for t is one minute per unit. Table 2 presents part of the results that corresponds to the 8:50 hrs peak for the first class.

Table 2 Estimated results of the early and late arrival models

Day	T1 _	Parameters and Constants Early Arrival							
Day	11	CUE	е	e/c e/c	t* t*(hrs)	<u>x_dot(t*)</u>	U/c		
Monday	8 50	0.0597	1.016	0124 - 0629	74 53 8 44 31 AM	78.73	117.26		
Tuesday	8 50	0.9673	2 2 6 7	0 168 - 0 428	80 00 8 49 59 AM	35.29	120 00		
Wednesday	8 50	5 5261	1.646	0102 - 0216	75 53 8 45 31 AM	48 59	117.77		
Thursday	8 50	2 0297	1.298	0102 - 0052	73 30 8 43 18 AM	61.65	116 65		
Friday	8 50	0.4022	1.202	0120 - 0198	72 34 8 42 20 AM	66 57	116 17		

	_		Parameters and Constants Late Arriva						
Day	T1	CUL	ı	I/d	I/d	t*	t*(hrs)	<u>x_dot(t*)</u>	U/d
Monday	8 50	- 25529.1	1.863	0.006 -	0 104	74.53	8 44: 31 AM	42 93	29 48
Tuesday	8 50	- 255290	1.997	0.006 -	0.065	80 00	8 49 59 AM	40.05	3480
Wednesday	8 50	- 254191	2 2 3 0	0.007 -	Q 107	75 53	8 45 31 AM	35.87	29 69
Thursday	8 50	- 254191	2 0 2 2	0.006 -	0 118	73 30	8 43 18 AM	39 56	27.98
Friday	8 50	- 254191	2016	0.006 -	0.125	72 34	8 42 20 AM	39 68	27.12

As seen on Table 2 all estimates for the arrival model hold the expected signs. For example, as time increases the disutility generated by the early cost of departure decreases, in other words e < 0 for early arrivals. The same happens with the late arrivals, that is l < 0. As for the disutility generated by the delay of the flow and the trouble of walking through the parking lot, the parameters  $\lambda$  and  $\kappa$  are positive.

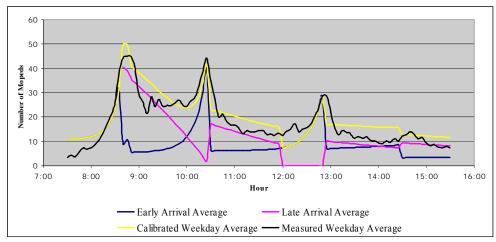


Figure 4 Weekly averages of the commuting flow and its reproduction.

The early arrival and late arrival flows and their comparison with the measured traffic are presented on Figure 4. It can be seen that the composition of the average calculated flow is the sum of both early arrival and late arrival calculated models. From this Figure 4, it can also be seen how the peaks are more concentrated when the early arrival flow is calculated.

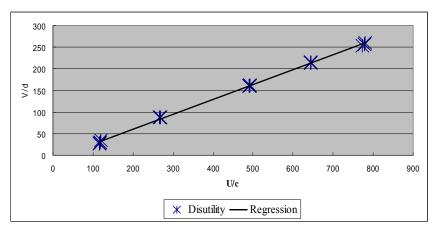


Figure 5 Disutility for early and late arrivals.

Table 3 Estimated disutilities for each class

Day	T1		T2		Т3		T4		T5	
	U/c	V/d	U/c	V/d	U/c	V/d	U/c	V/d	U/c	V/d
Monday	117.26	29.48	266.69	87.49	490.10	160.10	643.50	213.50	773.00	253.00
Tuesday	120.00	34.80	267.19	87.49	491.49	160.12	643.82	213.82	780.00	260.00
Wednesday	117.77	29.69	267.19	87.49	492.37	160.13	644.06	214.06	777.56	257.56
Thursday	116.65	27.98	267.50	87.50	491.74	162.37	644.88	214.88	777.56	257.56
Friday	116.17	27.12	267.76	87.50	490.08	160.08	643.26	214.88	777.56	257.56

From the arrival model, the relationship between  $\overline{U}/c$  and  $\overline{V}/d$  was obtained from the results of the disutility seen on Table 4. There are slight variations since the number of classes and their attendants, who are the student commuters, vary from day to day. For example, Tuesday disutility level seems higher than the other types of days because Tuesday is more congested due to more classes.

These results are plotted in Figure 5. From this figure it can be seen that the disutility grows in

a linear manner but also that the disutility for each day is close to each other during weekdays. Moreover, the relationship between c/d was calculated (the slope of the regressed line in Figure 5), it is 3.2 times more costly to arrive late than early.

## 4.2 Parameter Estimation for the Departure Model.

Based also on the traffic counting by weekday, seen on Figure 6, the moped exiting flow occur after the classes finish. We considered in this work that the students start to return home at 12:00 hrs, after the second class finish. The same method from the arrival model was used in the departure model. Also, a part of the estimation results is presented in Table 4. This part corresponds to the peak starting at 12:00 hrs, which is to the finish time of the second class or  $TF_2$ .

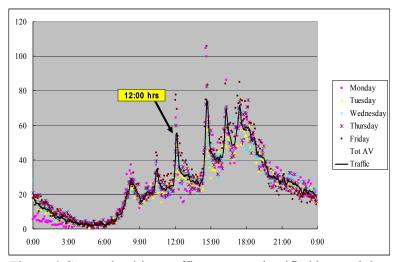


Figure 6 Counted exiting traffic patterns classified by weekday.

Based on the results seen on Table 4, parameter  $\delta$  didn't make the expected sign. In this sense  $\delta$  holds the sign, which can be interpreted, as the more exiting flow existing is easier to depart from class. All other parameters like waiting time on campus before departure or the discomfort of having parked afar hold the expected sign.

Table 4 Estimated results of the late departure model

Dox	TF2	Parameters and Constants Late Departure							
Day	1172	IC	Ψ	δ	3	ı			
Monday	12:00 hrs	389.824	103.960	-5.515	211.498	2374.510			
Tuesday	12:00 hrs	710.521	103.280	-20.787	220.435	2371.213			
Wednesday	12:00 hrs	612.516	95.155	-14.978	210.715	2374.968			
Thursday	12:00 hrs	991.438	106.071	-17.152	210.894	2374.520			
Friday	12:00 hrs	1233.462	89.152	-10.011	213.343	2375.064			

Changes also occur day by day, similar to the arrival model, Tuesday has more classes and a large group students as its attendants depart after class. This explains the difference in the parameter  $\delta$  in Table 4.

For the departure model, using the estimated parameters shown in Table 4, the comparison between the measured exiting traffic data and the calculated data was done and plotted on Figure 7. Also, from Figure 7 in can be seen there is some difference between the peaks calculated and the counted raw data. This is explained with the placement of the measurement

equipment and the time of departure. In other words, it takes some time to reach the traffic data counter when exiting the class and that student being registered by the traffic data counter.

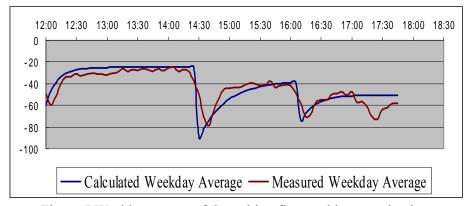


Figure 7 Weekly average of the exiting flow and its reproduction.

#### **5 CONCLUSIONS**

In this research we have proposed models that describe the moped flow when there are specific target points in time. Arrivals and departures are described in terms of disutility and the equilibrium conditions. Road traffic data were collected and used to validate the models. The proposed validation procedure suggests good stability in the model when finding the parameter estimates and a good reproduction performance. In some cases, the observations were not well reproduced; most of the cases are when the weekday had an unusual traffic flow pattern. The validation assures that the model performs well in similar context and can be used for forecasting. Also, these models can be used to measure the impacts of new policies regarding commuting to the University Campus.

The proposed model, while focusing on student behavior commuting to a university campus, can be used to other situations when similar conditions are met. For example, in order to reach a specific point in time, such as concerts, movies or theatrical performances and sport events, the model might be adapted to these particular circumstances. In this proposed model, during validation for arrivals, conditions for describing early and late arrivals were done. Meanwhile, only late departures where calculated. In this sense, if similar conditions are met such as a sporting event, this might imply that until the event ends, there occur no departures.

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