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Network Fortification Model for Intercity Passenger Transportation

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Abstract: This paper proposes a mathematical planning model to find the most effective links to be fortified, in order to secure the inter-city passenger transportation service facing to interdiction risk of each link in the network. If interdicted, the passengers must find a minimum time detour route and endure additional travel time. We assume that fortification of a link can avoid the risk of interdiction. The planning model is formulated based on the Multi-modal Network Planning (MNP) model formulated by the authors. This paper shows the model formulation and case study on a simple network including the introduction of emergent airline service after interdiction of railway links, as an illustration of the potential of the model.

Keywords: Fortification, Intercity, Network, Railway, Disaster

1. INTRODUCTION

Securing essential intercity transportation service is one of the important conditions to swift recovery from a large disaster or a destructive accident. For example, the Japanese government settled 4 basic objectives in the Basic Plan of National Resilience in June 2014; In any disasters, the Government must (1) secure the people's life as much as possible, (2) maintain the essentially important functions of the Japanese Government and Japanese society, (3) minimize damages in public infrastructure and people's properties, and (4) realize a swift recovery and restoration. The plan illustrates several situations to be avoided, such as disconnections in important trunk land-sea transportation route along the Metropolis Axis, concurrent loss of plural airports, appearance of isolations of several areas, and loss of transportation function causes delay of recovery process, therefore show the importance of swift setting up of alternative transportation routes in both locally and nation-widely. Based on the experience of the Great East Japan Earthquake (GEJE) case in 2011, the plan insisted the importance of multi-modal alternative route, adding to the preparation of alternative routes inside each transport mode.

This paper proposes a mathematical model to find the links to be fortified which efficiently increase the resilience of intercity passenger transportation network facing the disconnection risk due to natural disasters or accidents. The model can consider usage of multimodal route including intermodal transfers inside, if any single mode is not available between origin and destination. This flexible multimodal route can be found by the authors'

Multi-modal Network Planning (MNP) model and we add selecting part of fortifications to it. The proposed planning model is formulated as a Mixed Integer Linear Programming model, for which, swift commercial solver programs are available.

Our interests are what is the essential criterion to select effective links to be fortified. Is it the traffic volume in ordinal situation? Or is it the increase of travel cost by cutting each link? Further, does such selection become different, if we can use multi-modal service such as temporary flights or express bus service? Needless to say, answers to these questions above are strongly dependent on the network settings and parameters. This study aims to provide a prototype model to investigate these questions, generally. If we need not consider the minimum frequency in each link, or we focus on a single mode network fortification, we can reduce the number of variables and that of equations; it may ease larger number of trial calculations under various settings. Such simplifications of the proposed model can be possible easily.

The paper is organized in 5 sections. After the introductory section above, the following section reviews previous studies on estimation of stoppage effect of regional infrastructure and fortification planning. Section 3 formulates a mathematical planning model of effective fortification in the intercity passenger transport network. Section 4 illustrates the implications of the model, based on case study results on a simple network. At last, section 5 concludes the findings and shows the future works.

2. EXISTENT STUDIES ON STOPPAGE EFFECT OF INFRASTRUCTURE AND FORTIFICATION PLANNING

2.1 Stoppage of Local Facilities and Fortification

Studies on stoppage of infrastructure began to consider firstly on the local facilities providing service to the residents nearby. Representative planning problems are the n-Median problem which finds the location of *n* local facilities minimizing the total access distance of the users to their nearest service facility, and the n-Center problem which finds the location of *n* facilities minimizing the longest access distance from all demand points. Church *et al.* (2004) defined r-interdiction pattern as the *r* facilities lose their function at once and formulated the problem to find the most harmful r-interdiction pattern in the n-median and n-center problems as mixed integer linear programming model. Church and Scaparra (2007) defined a prior fortification as beforehand reinforcement of the selected facility making free from lost of function if it is interdicted. Their paper formulated the problem to find the most effective set of *p* fortifications out of *n* facilities.

In the p-fortification problem, there are two different assumptions for interdiction pattern. One is the case when we can estimate probability (either objective or subjective) of each interdiction pattern; we can use the expectation value of the interdiction effect based on the probability-weighted sum of effects. It is applicable to natural disaster cases, for example. The other case is when we have no prior knowledge for the probability, and then we must expect the most harmful situation. The planning problem to find the fortification pattern is formulated as two-level problem with mini-max structure. It is applicable to the fortification versus terrorists' attacks. Further, Gilboa and Schmeidler (1989) had generally insisted that rational choice of human facing to uncertain situation can be well described by a combination of the upper two cases; maximization of weighted sum of utility in each possible situation and maxi-min utility (or mini-max cost) optimization.

Scappara and Church (2008) formulated a mini-max problem as two-level mixed integer linear programming problem. As Choi and Suzuki (2013), this two-level problem can be translated into one-level linear programming problem by defining the state variables on interdiction patterns. But this translation results increase of problem size and longer computation time.

These studies describe the interdiction or stoppage of each facility using a discrete {0, 1} variable, and so for fortification or protection. Both number of interdictions (*r*) and fortifications (*p*) are externally given as an integer number. However more flexible models have been suggested; Liberatore *et al.* (2011) considers the probability of each facility interdiction but not assume the number of concurrent interdictions; Zhu *et al.* (2013) models probabilistic effect of fortification. These formulations however, scarify the uniqueness of the optimal solution, and require heuristic approach instead of reliable mathematical programming methods.

2.2 Stoppage and Fortification of Transportation Network

There are enormous studies on the effect of stoppage in transportation network compiled in the field of network reliability analysis, as summarized by Kurauchi *et al.* (2007). For the quick response to the disaster or accident, they pay attention on reachability within a given time and isolation of nodes. They define reliability of connection and applied to discuss whether rescue team can arrive to the affected area in certain time or not. Several days after, it became important to check the performance of the survived network to provide effectively for the ordinal transportation demand pattern. They may evaluate the increase of travel time and cost by detours on the survived portion of the network, sometimes consider congestion or capacity limit due to the route alteration of the traffic in a flow-dependent framework.

Bell *et al.* (2008) and Cappanera and Scaparra (2011) proposed a two-level planning model to obtain an efficient order of link fortifications and discussed solution algorithms.

2.3 Fortification of Intercity Passenger Transportation Network

In the contrast with the thick accumulation of studies in local area network reliability and fortification problem as illustrated above, academic researches on national level network fortification are very scarce, other than Taylor *et al.* (2006). Especially, intercity public passenger transportation after disaster has not yet gathered any attention of researchers or policy-makers, in contrast with inter regional freight transportation after disaster.

The Japanese experience in the 2011 disaster taught us the importance of passenger transport between the affected area and the national capital city (Tokyo), in the recovery and restoration process. Because of huge scale of the disaster, the recovery work cannot be planned and decided by the local government or local branch offices of the national government in the affected area only. Not a few experts and policy makers in the other area must enter the disaster area to inspect the damages and available local resources for recovery. Comparing to the 100 days recovery of Shinkansen service in 1995 Kobe Earthquake, Shinkansen system was free from structural breakdowns of bridge or viaduct in 2011 GEJE. The quicker recovery in 50 days helped the acceleration of the recovery process, as well as the temporally provided flight services to Yamagata Airport and Hanamaki Airport from Tokyo. We learnt the importance of the continual intercity passenger transportation service. The main problem will be the fortification of High Speed Railway network, but we should consider the temporal service of other travel modes, such as domestic flights and expressway buses.

3. FORTIFICATION PLANNING MODEL BASED ON MNP MODEL

The present authors formulated a Multimodal Network Planning (MNP) model to investigate the optimal intercity passenger transport network providing shorter travel time and smaller operation cost ensuring enough capacity for the given OD passengers, in Okumura *et al.* (2012). The Fortification planning model is formulated by adding the part to determine the effective fortification set of links, to the MNP model. In this section, we describe the formulation of the MNP as well, in order to explain the fortification model structure.

3.1 Basic Structure of the MNP Model

The MNP model distributes the given OD traffics onto a multi-modal transport network and find the frequency of all links in the network that minimize the monetary sum of total passenger travel time and total operation cost. This model was formulated as a mixed integer linear programming model. In the problem, we ensure the feasibility of operation and capacity limit for each link; total passengers using that mode in a given link must sit between the upper capacity limit (number of the seats provided) by the frequency and the lower limit related to the minimum operation cost coverage by the fare from the passengers.

In the fortification model to be considered here, we assume that operation cost just after the disaster will be financed by special budget of national government, then consider the total travel time including detours under the physical connectivity and capacity. Sensitivity analysis of this model can provide the tradeoff information between total travel time and the cost of fortification and additional operations.

3.2 Endogenous Variables and Model Parameters

In our network design, each city is represented by a node $n \in N$ and connecting arcs between two cities through different modes $m \in M$ are indicated by $(i,j) \times m \in L$. M and L indicate the set of modes and nodes, respectively. In order to express transit connection between modes explicitly, each node is divided into arrival node by mode as $n_{-} \times m$ and departure node by mode as $n_{+} \times m$, and transit arc indicated by $(m, m^{'}) \times n$. Also, amount of OD traffic between zones $(k,l) \in K \times K$ is given by T_{kl}, exogenously. K indicates the set of centroid zones.

We call the set of links losing their function as "Interdiction Pattern" and denoted it by index $h \in H$. In this study, we assume two cases either when we can estimate the probability of occurrence q^h based on historical records or scientific risk assessment, or when we cannot know them. For each interdiction pattern, we denote the stoppage of each modal link by {0, 1} variable P_{ij}^{mh}. Further, we assume that prior fortification or reinforcement can decrease the effect of interdiction to the link capacity as ρ_{ij}^{m}. Our decision variable is {0, 1} variable R_{ij}^{m} indicating whether each link is fortified or not.

We consider traffic flow variables and set frequency as other endogenous variables indexed by interdiction pattern $h \in H$. Four traffic flow variables are as follows. X_{ij}^{kmh} indicates the traffic on the ordinal transportation link of mode m from origin k. $Y_{n}^{kmm'h}$ indicates intermodal transfer traffic at node n, originated from k. B_{k}^{mh} is defined as traffic originated at node k beginning their trip by mode m. At last, A_{n}^{kmh} means the finalizing traffic at node n through mode m. Modal frequency of service is denoted as F_{ij}^{mh}.

Exogenously given parameters are as follows; t_{ij}^{m} indicates travel time by vehicle of mode m through each link. $\tau_n^{mm'}$ indicates required time to transfer at each node, and we don't distinguish the transfer to the same mode with through service beyond the node. s_m gives passenger number of one frequency service, such as train or airplane seat capacity. g_m indicates the maximum frequency of operation for each mode. In addition to the above parameters, we consider fortification cost of each link c_{ij}^{m}, and total fortification budget C, as exogenous parameters.

3.3 Two Types of Objective Function

We can formulate two types of objective function based on total travel time of passengers, corresponding to the different decision rule for uncertainty. The first formulation is the case when we can estimate the probability of each interdiction pattern q^h. In this case we simply use the expectation value of total travel time, given by a weighted sum of travel time in each interdiction cases. It can be formulated as a linear function of the endogenous variables as follows:

$$\min_{R_{ij}^{m}} ET = \sum_{h\in H} q^h \left(\sum_i \sum_j \sum_m t_{ij}^{m} \sum_k X_{ij}^{kmh} + \sum_n \sum_m \sum_{m'} \tau_n^{mm'} \sum_k Y_n^{kmm'h}\right). \quad (1)$$

The two terms in the parentheses in the RHS of eq. (1) denote total travel time in vehicle and the total intermodal transfer time, respectively.

The second formulation is mini-max objective function, which minimizes the most harmful interdiction pattern to the total travel time. This is the case when we cannot know the probability of each interdiction q^h, or terrolist attack case. It can be rewritten as the pair of the two following formula:

$$\min_{MT, R_{ij}^{m}} MT. \quad (2)$$

$$MT \geq \left(\sum_i \sum_j \sum_m t_{ij}^{m} \sum_k X_{ij}^{kmh} + \sum_n \sum_m \sum_{m'} \tau_n^{mm'} \sum_k Y_n^{kmm'h}\right), \forall h \in H \quad (3)$$

MT in eq. (2) and (3) indicates the maximum total travel time, and eq. (3) ensures that MT is not inferior to the total travel time under any interdiction pattern.

Either objective function, we must add a budget constraint for fortification decision R_{ij}^{m} as follows:

$$\sum_i \sum_j \sum_m c_{ij}^{m} R_{ij}^{m} \leq C. \quad (4)$$

3.4 Physical Conditions for Traffic Variables

Conservation conditions are formulated as linear constraints. At an arrival node n, the following two conditions are required.

$$\sum_{i \in N^-(n)} X_{in}^{kmh} = A_n^{kmh} + \sum_{m' \in M} Y_n^{kmm'h} \; \forall n \in N, \forall k \in K, \forall m \in M, \forall h \in H, \quad (5)$$

$$\sum_m A_n^{kmh} = T_n^k, \;\; \forall n \in N, \forall k \in K, \forall h \in H. \quad (6)$$

Eq. (5) describes that traffic arrived at node n by travel mode m will be divided into the passengers with node n as final destination, and the passengers to use same mode or transfer from that mode for further travels. Eq. (6) indicates that the sum of the passengers finalizing their trip at that node by all modes must be equal to the given OD traffic demand.

Similarly, the following set of equations is given for the passengers departing from node n.

$$B_n^{mh} + \sum_{m' \in M} Y_n^{km'mh} = \sum_{j \in N^+(n)} X_{nj}^{kmh} \;\; \forall n \in N, \forall k \in K, \forall m \in M, \forall h \in H \quad (7)$$

$$\sum_{l \in N} T_{nl} = \sum_{m \in M} B_n^{mh}, \;\; \forall n \in K, \forall h \in H. \quad (8)$$

Eq. (7) describes that both of passengers originating their trip at that node by mode m and passengers transferred into mode m generate the traffic departing from that node. Eq. (8) ensures that total OD traffic originating node n can find a certain first travel mode.

We formulate link capacity condition, next. We can set positive frequency F_{ij}^{mh} on mode m on the link up to predetermined capacity g_m, either when that link did not get interdiction ($P_{ij}^{mh} = 0$), or when that link had been fortified ($R_{ij}^m = 1$). This condition is given by the following two equations.

$$F_{ij}^{mh} \leq g_m(1 - P_{ij}^{mh} + \rho_{ij}^m R_{ij}^m), \;\; \forall (i,j) \times m \in A, \forall h \in H. \quad (9)$$

$$F_{ij}^{mh} \leq g_m, \;\; \forall (i,j) \times m \in A, \forall h \in H. \quad (10)$$

Link capacity parameter g_m can be given differently for link by link, considering physical conditions or resource availability.

Relationship between the set frequency and total link traffic is given as follows:

$$\sum_k X_{ij}^{kmh} \leq s_m F_{ij}^{mh}, \forall (i,j) \times m \in A, \forall h \in H. \quad (11)$$

3.5 Closing the Formulation and the Solving Method

At last, we add definition space for the endogenous variables:

$$X_{ij}^{kmh} \geq 0, Y_n^{kmm'h} \geq 0, A_n^{kmh} \geq 0, \ B_n^{mh} \geq 0, F_{ij}^{kmh} \geq 0 \tag{12}$$

And that for the decision variable:

$$R_{ij}^m \in \{0,1\} \tag{13}$$

Now we can summarize the formulation of the two models. Minimization of the expected total travel time given by eq. (1) under the constraints of eq. (4) to (13), and Mini-max problem minimizing eq. (2) under the constraints of eq. (3) to (13). Both problems are formulated as a mixed integer linear programming problem including small number of {0, 1} variable R_{ij}^m. This type of problem can be solved by any calculation package provided for most mathematical software platforms.

4. A SIMPLE NETWORK ILLUSTRATION

4.1 Simple Hypothetical Network

The model formulated in section 3 can be applied to a multimodal network, but here in order to show the potential of the proposed model, we consider a simple network consisted by 13 links of railway mode only, shown as blue lines in Figure 1. Travel time of all links is constantly set as $t_{ij}^{rail} = 50$ (minutes). Further we assume that new temporal air service will be provided between the cities possessing airport as shown as dotted red arcs in Figure 1. Flight time is set as long as $t_{ij}^{air} = 120$ (minutes). This setting means that flight service will be used only when people need a detour more than 3 railway links. Transfer time at each node is given as, $\tau^{rail,rail} = 15$, $\tau^{rail,air} = \tau^{air,rail} = \tau^{air,air} = 60$ (minutes). It means that rail-air transfer occurs only when the parallel railway service is not available. Seat capacity of one frequency of rail and flight are given as, $s_{rail} = 400$, $s_{air} = 200$ (passengers/operation), respectively. Frequency limit for each link is given as $g_{rail} = 50$, $g_{air} = 50$ (operations), large enough to set freely if needed. Travel demand for all OD pairs are constantly given as $T_{nl} = 200$ (passengers), regardless the distance.

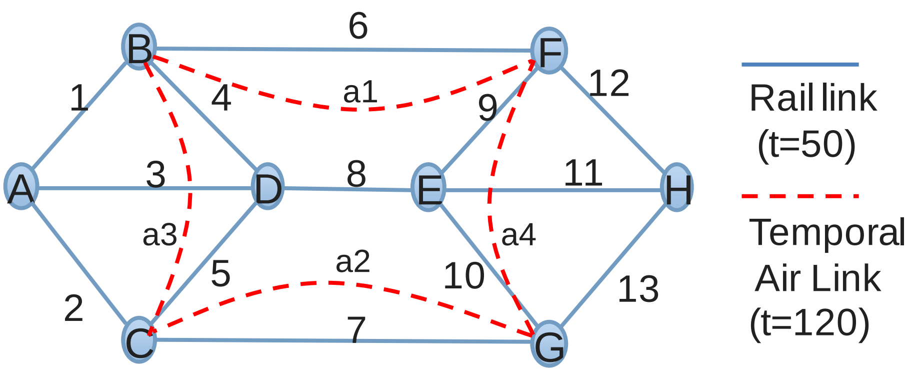

Figure 1. Hypothetical transport network

4.2 Estimate the Effect of Stoppage of Railway Service

In order to find the effect of each case of interdiction, we set no fortification budget (C=0) and solve the first model minimizing the expected total travel time for given interdiction pattern. Here we consider the two different settings of interdiction patterns. The first setting is that one of 13 links is interdicted with same probability, 1/13. The second setting is that any two links from 13 links are evenly selected and interdicted: ${}_{13}C_2 = 78$ combinations. It took 6 hours to complete using lp_solve library with C++ on a PC with Intel Xeon(R)E5-2609V2@2.50GHz, 32 GB ram and 200GB free space to complete the 78 combinations. Because most variables are defined with the superscript h, it is indispensable to reduce the number of the interdiction patterns based on a careful disaster risk assessment, when we should apply the model to the realistic size network.

Table 1 shows the total travel time and the required frequency of each link in each interdiction case. Under the given parameter values, twice of the required frequency gives the number of OD pairs using the corresponding link. According to the traffic in the full network (no interdiction), central links connecting east and west {7, 8} are mostly used, followed by link 2. Links {1, 5, 6, 9} are used for 4 OD pairs, followed by links {3, 4, 11, 13}. Links {10, 12} are used for 2 OD pairs, only. The effect of single interdiction cases seems similar to the ordinal time traffic, but the orders of them are not identical. All of three center links {6, 7, 8} have the same effect of 104, 000 minutes increase. Links at east and west edges {1, 2, 3, 11, 12, 13} have the second largest effect of 52,000 minutes increase. Comparing to those, links {4, 5, 9, 10} have smaller effect of 26,000 minutes increase.

Table 1. Impact of interdiction on travel time

Destruction Set		**Line Frequencies**												
Destructed Links	**Travel time**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
none	1,028,000	2.0	2.5	1.5	1.5	2.0	2.0	3.0	3.5	2.0	1.0	1.5	1.0	1.5
1	1,080,000	0.0	1.5	2.5	4.0	3.0	3.0	3.0	3.0	1.0	1.0	1.5	1.5	2.0
2	1,080,000	1.0	0.0	2.5	2.0	4.0	1.5	2.5	4.5	2.0	1.5	1.5	1.0	1.0
3	1,080,000	2.0	3.5	0.0	1.0	3.0	1.5	3.5	3.5	2.0	1.5	1.0	1.0	1.5
4	1,054,000	3.0	2.5	2.0	0.0	2.0	3.0	2.5	3.0	2.5	1.0	1.5	1.5	1.5
5	1,054,000	2.5	3.0	2.0	1.5	0.0	2.0	3.0	4.0	2.0	3.0	1.5	1.0	1.0
6	1,132,000	1.5	2.5	2.0	2.0	2.0	0.0	3.0	5.5	3.0	1.0	2.0	0.5	1.5
7	1,132,000	2.0	1.5	2.0	1.5	2.5	2.0	0.0	6.5	2.0	3.0	2.0	1.0	0.5
8	1,132,000	2.0	3.0	0.5	1.5	2.5	3.0	5.0	0.0	2.0	3.0	0.5	1.0	2.0
9	1,054,000	2.0	2.0	1.5	2.0	1.5	3.0	3.0	3.0	0.0	1.0	2.0	3.0	2.5
10	1,054,000	2.0	2.5	1.0	1.0	2.0	1.5	3.5	3.5	1.5	0.0	1.5	2.0	3.0
11	1,080,000	2.0	2.5	1.0	1.0	1.0	2.0	3.0	3.0	1.5	2.5	0.0	1.5	3.0
12	1,080,000	2.5	2.5	1.5	1.5	1.5	3.0	3.5	3.5	4.0	2.5	2.5	0.0	1.5
13	1,080,000	2.0	2.0	1.5	1.0	2.5	1.5	3.0	4.5	2.0	2.5	2.5	1.0	0.0
(6,8) (7,8) (max)	1,444,000	3.0	5.0	0.5	2.0	4.0	0.0	8.0	0.0	3.0	5.0	0.5	0.5	2.5
(4,5) (4,10) (5,9) (9,10) (min)	1,080,000	3.0	3.0	2.5	0.0	0.0	3.0	2.5	3.0	2.5	2.5	1.5	1.0	1.0

The last two lines in Table 1 shows the effect of two link interdictions. Most serious interdiction patterns are when any two of the east-west links in the center {6, 7, 8} are interdicted at once, then give four times of the single interdiction of the link. According to the result shown in the last row in Table 1, most of combinations of the less important links {4, 5, 9, 10} gives simply double of the single interdiction effect (52,000 minutes).

4.3 Optimal Fortification Minimizing the Expected Travel Time

At first, we assume single link interdiction with even probability, $q^h = 1/13$. Prior fortification is assumed to give perfect security of service, that is $\rho_{ij}^{rail} = 1$. In this case, the effect of fortification appears when the same link is interdicted with probability, 1/13. If we consider the budget of fortification as much as one link fortification, the expected effect of fortification is given by the effect shown in Table 1, divided by 13. It means, one link in {6, 7, 8} is the most efficient fortification. If we have more fortification budget, we select the links following the order of single interdiction effects shown in Table 1.

Next, we consider 2 link interdiction cases, $q^h = 1/78$. We assume p links can be fortified by setting $C = pc_{ij}^{rail}$. Table 2 shows the result of solution of the minimizing the expected value of total travel time, by changing p from 2 to 13. Table 2 also shows the worst interdiction set and total travel time for the network after fortifications. Obtained fortification order is almost similar with the order of single interdiction effect; {6, 7, 8} in the center,{1, 2, 3, 11, 12, 13} on the edges, and at last {4, 5, 9, 10} in the middle.

Table 2. Optimal fortification strategy (for r = 2)

Results				**Worst Case**	
r	**p**	**Mean trav. time**	**Protected links**	**Destruction set**	**Travel time**
2	2	1,122,000	6,7	(1,4) (2,5) (9,12) (10,13)	1,236,000
2	3	1,105,333	6,7,8	(1,4) (2,5) (9,12) (10,13)	1,236,000
2	4	1,094,667	6,7,8,1	(2,5) (9,12) (10,13)	1,236,000
2	5	1,084,000	6,7,8,1,2	(9,12) (10,13)	1,236,000
2	6	1,073,333	6,7,8,1,2,12	(10,13)	1,236,000
2	7	1,062,667	6,7,8,1,2,12,13	(3,4)(3,5)(3,11)(4,9)(5,10) (9,11)(10,11)	1,132,000
2	8	1,054,000	6,7,8,1,2,12,13,3	(4, 9)(5,10) (9,11) (10,11)	1,132,000
2	9	1,045,333	6,7,8,1,2,12,13,3,11	(4, 9) (5,10)	1,132,000
2	10	1,040,667	6,7,8,1,2,12,13,3,11,4	(5, 10)	1,132,000
2	11	1,036,000	6,7,8,1,2,12,13,3,11,4,5	(9, 10)	1,080,000
2	12	1,032,000	6,7,8,1,2,12,13,3,11,4,5,9	(*, 10) (...)	1,054,000
2	13	1,028,000	6,7,8,1,2,12,13,3,11,4,5,9,10		1,028,000

Furthermore, we calculated similarly for 3-interdiction (${}_{13}C_3 = 286$) cases of the flat probability of $q^h = 1/286$, and the result is shown in Table 3. Because both node A and node

H in this network are connected by 3 links with other nodes, concurrent interdiction of three links resulted the isolation of one node. In order to avoid such isolations, at least two links connecting to both nodes must be fortified. In the solution, links {1, 12} are chosen at first.

Further fortification order is given as the central important links {6, 7, 8}, followed by edge links {2, 13}, and the other links {4, 5, 9, 10}, just similar to the single interdiction cases. But in $p = 6$, quit the fortification of link 12, and choose link 13 instead.

Table 3. Optimal fortification strategy (for r = 3)

Results				**Worst Case**	
r	**p**	**Mean trav. time**	**Protected links**	**Destruction set**	**Travel time**
3	3	1,193,818	1,7,12	(2,6,8) (5,6,8) (6,8,10) (6,8,13)	1,574,000
3	4	1,153,091	1,7,12,6	(2,5,8) (8,10,12)	1,496,000
3	5	1,125,455	1,7,12,6,8	(2,3,4)(2,5,10) (5,10,13) (9,11,13)	1,392,000
3	6	1,104,909	1,7,6,8,2,13	(4,9,12) (10,11,12)	1,392,000
3	7	1,084,364	1,7,6,8,2,13,12	(3,4,5) (9,10,11)	1,236,000
3	8	1,070,182	1,7,6,8,2,13,12,3	(9,10,11)	1,236,000
3	9	1,056,000	1,7,6,8,2,13,12,3,11	(4,5,9) (4,5,10) (4,9,10) (5,9,10)	1,158,000
3	10	1,048,000	1,7,6,8,2,13,12,3,11,4	(5,9,10)	1,158,000
3	11	1,040,000	1,7,6,8,2,13,12,3,11,4,5	(9,10,*) (..)	1,080,000
3	12	1,034,000	1,7,6,8,2,13,12,3,11,4,5,9	(10,*,*) (..)	1,054,000
3	13	1,028,000	1,7,6,8,2,13,12,3,11,4,5,9,10		1,028,000

4.4 Mini-max Fortification of Railway Network

Now we compare the alternate objective function, for the 3 links interdiction patterns. Table 4 shows the solutions of the mini-max problem minimized eq. (2). At first links {1, 12} are selected in order to avoid the isolation of the edge zones A, H. Fortifications of the important east-west links {7, 8} are followed, as the previous problem. Subsequent fortifications become different; at $p = 5$, links {2, 13} at both edges are selected with forgiving link 7. At $p = 6$, the central east west links {6, 7} are selected to complete the outer circle reinforcement, rather than the central link 8. We can find the alternations of the selections occur also at p=8 and p=11.

Orders of reinforcement seem unstable, compared to the expected travel time minimization. This comes from that different interdiction patterns can change directly the efficient links of fortification and the mini-max solution. But such alternations were leveled by taking the expectation value in the first objective function, eq. (1).

Table 4. Mini-max fortification strategy (for r = 3)

Results				**Worst Case**	
r	**p**	**Mean trav. time**	**Protected links**	**Destruction set**	**Travel time**
3	3	1,193,818	1,7,12	(2,6,8) (5,6,8) (6,8,10) (6,8,13)	1,574,000
3	4	1,158,000	1,7,8,12	(2,3,4)(2,5,10) (5,10,13) (9,11,13)	1,392,000
3	5	1,137,181	1,2,6,8,13	(3,5,7)(4,9,12) (7,10,11) (10,11,12)	1,392,000
3	6	1,110,909	1,2,6,7,12,13	(4,5,8) (4,9,10)	1,288,000
3	7	1,084,363	1,2,6,7,8,12,13	(3,4,5)(9,10,11)	1,236,000
3	8	1,093,000	1,3,5,6,8,9,10,11	(2,4,7)(2,7,12) (4,7,12) (7,12,13)	1,210,000
3	9	1,064,000	1,2,3,6,7,8,9,10,11	(4,12,13) (5,12,13)	1,158,000
3	10	1,052,000	1,2,3,6,7,8,9,10,11,12	(4,5,13)	1,132,000
3	11	1,040,000	1,2,3,4,5,6,7,8,11,12,13	(*,9,10) (..)	1,080,000
3	12	1,034,000	1,2,3,4,5,6,7,8,9,11,12,13	(10,*,*) (..)	1,054,000
3	13	1,028,000	1,2,3,4,5,6,7,8,9,10,11,12,13		1,028,000

4.5 Effects of Temporal Air Links

Let us investigate the effects of temporal air service. We use the first model of expected total travel time minimization model applied for 3 rail links interdiction situations.

At first, we assume the availability of air link a1 in Figure 1, after the railway interdiction. Table 5 shows the result adding the most right column showing the frequency of the temporal air link in the worst attack pattern after the p rail link fortification. In comparison with Table 3, the existence of air link a1 decreased the importance of link 5 and fortifications occur in links {2, 13, 7} locating the other side of the network. After the reinforcement of the rail link 5 at $p = 5$, the parallel more time-consuming air link a1 is not used anymore. Fortification pattern at $p = 6$ is symmetrical to the case in Table 3, giving the identical time value. Further solutions after $p = 7$ are identical to Table 3.

Secondly, Table 6 shows the solutions when two air links are available. At $p = 2$, the connections to the edge zone avoiding isolation are done by the links {1, 12}, which are directly connected with the air links. Link 8 without parallel air service is selected in the three east-west links at $p = 3$. After $p = 5$, the solution is identical with Table 3, which has no air service.

Table 7 and Table 8 show the result when 3 or 4 air links are available, respectively. We can find the difference of fortification orders in number of fortification p is small.

Table 5. Optimal fortification strategy (in the case of special air link = 1)

Best Protection				**Worst Case**		
r	**p**	**Mean trav. time**	**Protected links**	**Destruction set**	**Travel time**	**Freq. a1**
3	2	1,224,266	2,13	(6,7,8)	2,324,000	16
3	3	1,180,657	2,13,7	(1,4,6) (6,9,12)	1,540,000	7
3	4	1,148,140	2,13,7,8	(1,4,6) (6,9,12)	1,540,000	7
3	5	1,125,455	7,8,1,6,12	(2,3,4) (2,5,10) (5,10,13) (9,11,13)	1,392,000	0
3	6	1,104,909	7,8,1,6,12,2	(5,10,13) (9,11,13)	1,392,000	0
3	7	1,084,364	7,8,1,6,12,2,13	(3,4,5) (9,10,11)	1,236,000	0
3	8	1,070,182	7,8,1,6,12,2,13,3	(9,10,11)	1,236,000	0
3	9	1,056,000	7,8,1,6,12,2,13,3,11	(4,5,9) (4,5,10) (4,9,10) (5,9,10)	1,158,000	0
3	10	1,048,000	7,8,1,6,12,2,13,3,11,4	(5,9,10)	1,158,000	0
3	11	1,040,000	7,8,1,6,12,2,13,3,11,4,5	(*,9,10) (..)	1,080,000	0
3	12	1,034,000	7,8,1,6,12,2,13,3,11,4,5,9	(*,*,10) (..)	1,054,000	0
3	13	1,028,000	7,8,1,6,12,2,13,3,11,4,5,9,10		1,028,000	0

Table 6. Optimal fortification strategy (in the case of special air link = 2)

Best Protection				**Worst Case**			
r	**p**	**Mean trav. time**	**Protected links**	**Destruction set**	**Travel time**	**a1**	**a2**
3	2	1,211,524	1,12	(6,7,8)	1,904,000	10	6
3	3	1,171,664	3,8,11	(1,4,6) (2,5,7) (6,9,12) (7,10,13)	1,540,000	7	0
3	4	1,148,140	8,1,6,12	(2,5,7) (7,10,13)	1,540,000	0	7
3	5	1,125,455	8,1,6,12,7	(2,3,4)(2,5,10) (5,10,13) (9,11,13)	1,392,000	0	0
3	6	1,104,909	8,1,6,12,7,2	(5,10,13) (9,11,13)	1,392,000	0	0
3	7	1,084,364	8,1,6,12,7,2,13	(3,4,5) (9,10,11)	1,236,000	0	0
3	8	1,070,182	8,1,6,12,7,2,13,3	(9,10,11)	1,236,000	0	0
3	9	1,056,000	8,1,6,12,7,2,13,3,11	(4,5,9) (4,5,10) (4,9,10) (5,9,10)	1,158,000	0	0
3	10	1,048,000	8,1,6,12,7,2,13,3,11,4	(5,9,10)	1,158,000	0	0
3	11	1,040,000	8,1,6,12,7,2,13,3,11,4,5	(9,10,*) (..)	1,080,000	0	0
3	12	1,034,000	8,1,6,12,7,2,13,3,11,4,5,9	(10,*,*) (..)	1,054,000	0	0
3	13	1,028,000	8,1,6,12,7,2,13,3,11,4,5,9,10		1,028,000	0	0

Table 7. Optimal fortification strategy (in the case of special air link = 3)

Best Protection				Worst Case				
r	p	Mean trav. time	Protected links	Destruction set	Travel time	a1	a2	a3
3	2	1,205,797	3,12	(6,7,8)	1,892,000	2	10	6
3	3	1,164,573	3,8,11	(6,9,12)(7,10,13)	1,534,000	1	7	0
3	4	1,142,713	3,8,6,12	(7,10,13)	1,534,000	1	0	7
3	5	1,121,091	3,8,6,12,7	(5,10,13) (9,11,13)	1,392,000	0	0	0
3	6	1,100,455	3,8,6,12,7,13	(1,4,9) (2,5,10)	1,304,000	3	0	0
3	7	1,084,364	8,6,12,7,13,1,2	(3,4,5) (9,10,11)	1,236,000	0	0	0
3	8	1,070,182	8,6,12,7,13,1,2,3	(9,10,11)	1,236,000	0	0	0
3	9	1,056,000	8,6,12,7,13,1,2,3,11	(4,5,9)(4,5,10) (4,9,10) (5,9,10)	1,158,000	0	0	0
3	10	1,048,000	8,6,12,7,13,1,2,3,11,4	(5,9,10)	1,158,000	0	0	0
3	11	1,040,000	8,6,12,7,13,1,2,3,11,4,5	(*,9,10) (..)	1,080,000	0	0	0
3	12	1,034,000	8,6,12,7,13,1,2,3,11,4,5,9	(10,*,*) (..)	1,054,000	0	0	0
3	13	1,028,000	8,6,12,7,13,1,2,3,11,4,5,9,10		1,028,000	0	0	0

Table 8. Optimal fortification strategy (in the case of special air link = 4)

Best Protection				Worst Case					
r	p	Mean trav. time	Protected links	Destruction set	Travel time	a1	a2	a3	a4
3	2	1,199,608	3,11	(6,7,8)	1,892,000	2	10	6	0
3	3	1,157,566	3,11,8	(1,4,6) (2,5,7) (7,10,13) (6,9,12)	1,340,000	4	3	0	0
3	4	1,136,783	3,11,8,6	(2,5,7) (7,10,13)	1,340,000	4	0	3	0
3	5	1,116,364	3,11,8,6,7	(1,4,9)(2,5,10) (4,9,12) (5,10,13)	1,304,000	3	0	0	0
3	6	1,100,455	11,8,6,7,1,2	(4,9,12) (5,10,13)	1,304,000	0	0	0	3
3	7	1,084,364	8,6,7,1,2,12,13	(9,10,11) (3,4,5)	1,236,000	0	0	0	0
3	8	1,070,182	8,6,7,1,2,12,13,3	(9,10,11)	1,236,000	0	0	0	0
3	9	1,056,000	8,6,7,1,2,12,13,3,11	(4,5,9)(4,5,10) (4,9,10) (5,9,10)	1,158,000	0	0	0	0
3	10	1,048,000	8,6,7,1,2,12,13,3,11,4	(5,9,10)	1,158,000	0	0	0	0
3	11	1,040,000	8,6,7,1,2,12,13,3,11,4,5	(9,10,12) (..)	1,080,000	0	0	0	0
3	12	1,034,000	8,6,7,1,2,12,13,3,11,4,5,9	(1,8,10) (..)	1,054,000	0	0	0	0
3	13	1,028,000	8,6,7,1,2,12,13,3,11,4,5,9,10		1,028,000	0	0	0	0

5. CONCLUSION

This paper formulated an optimization model to find the link fortification pattern, most effectively avoid the effect of sudden stoppage of links in an intercity multimodal passenger transportation network, based on the authors' previous study on the MNP model. For the small-size hypothetical railway network, minimization of the expected total travel time model was applied. We found that the optimal fortification order is not always corresponding to the ordinary passengers' volume. It may rather correspond to the order of single interdiction to the total travel time. Mini-max problem minimizing the effect by the most harmful interdiction pattern gave the more unstable order of fortification. Furthermore, we investigated the effects of the temporal air service after interdiction on the fortification orders. We can conclude that our proposed model can provide a powerful tool to investigate the transport network reinforcement policies.

The results above are strongly dependent on the shape of the network and the given parameter settings. We need more number of calculations on several alternate settings. In order to do it, we must reduce the number of the variables through simplification of the problem setting.

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