Abstract: Public transportation service is essentially based on the scale economy, which comes from that cost of one vehicle operation can be divided by many passengers sharing that service. Therefore, it becomes very difficult to provide service in the area with low density of demand. Recently, in order to provide transportation service in such low density areas, demand responsive bus (DRB) gathers attention and expectations. This paper discusses the market externalities in DRB system in suburb and show what kind of fare arrangement will be needed to realize the efficiency of the market. At first, we analyze the optimal division of resources between two kinds of buses; demand responsive service and direct trunk line service. Without any fare intervention, people try to overuse of DRB because they neglect the external effects by their calls. Then we try to estimate the amount of the externalities and give information for the additional price for DRB.

Key Words: Demand responsive bus, Bus operation, Externality

1. INTRODUCTION

Traditionally, in order to connect a downtown and suburban housing complexes which locate away from the trunk road, a lot of direct bus lines are often provided. However, when the population decreases in the future, enough large volume of demand to maintain such direct bus service for each housing complex cannot be secured; any revisions of bus lines become inevitable. Then it is plausible that bus lines would be revised as some combination of the following services; direct trunk line bus which runs only along the trunk road, detour type bus service which goes around each housing complex while running at the trunk road, and the demand responsive bus (DRB) which goes around the housing complex according to the calls of the users (Suzuki, 2001).

The flexibility of the operation that DRB can skip running in the unnecessary part has been considered as an advantage characteristic of DRB system (Akiyama, T. et al., 2000). However, if someone calls a DRB, the external negative effects will occur, such as the increase of the required on vehicle time for the users who have already got on the bus at the outer bus stops, and the increase of waiting time of the users at bus stops in downstream where delay of arrival time occurs. In order to operate DRB efficiently, it is important to understand the characteristic of the operation form with paying attention to such negative external effects of bus calls on the other users.
This paper discusses the market externalities in DRB system and shows what kind of fare arrangement will be needed to realize the efficiency of the market. For computation example, a virtual bus system in suburb is considered. At first, we analyze the optimal division of resources between two kinds of bus; demand responsive service and direct trunk line service. Without any fare intervention, people may try to overuse DRB because they neglect the external effects by their calls. Then we try to estimate the amount of the externalities and give information for additional price for the DRB, which will guide the equilibrium more efficient.

2. DEMAND RESPONSIVE BUS OPERATION IN SUBURBS AND RELATED STUDIES UP TO NOW

2.1 Actual State of Demand Responsive Bus Operation

The necessity of the securing public transportation service is growing issue in Japanese rural areas. A lot of demonstration experiments in rural areas, represented by “Nakamura Machi Bus” in Nakamura City, Kochi Prefecture (Nakamura City, 2000), can be counted. In urban areas, however, few examples of DRB service can be enumerated, except the earliest case of the “Tokyu Coach” operated in Jiyugaoka area in Tokyo from 1976 and the circulating type of the DRB service by “Tokyu Transses” begun in 1998 in Daikanyama area in Tokyo.

The “Tokyu Coach” service was consisted of the combination of regular pre-determined service line and of DRB service detour line when it was called; both lines were reported to receive high evaluation (Nomura Research Institute, 1976). However, no less than 90% of bus vehicles got to run detours by the calls after 25 years from the beginning the DRB service, then DRB services were abolished, then to be unified as the regular operation line. At present, the DRB services in urban and suburb areas are very limited as the case of “Tokyu Transses” established by the Tokyo Bus Company in 1998, operating around Daikanyama area from July of the same year, and the case by the Toyama Chihou Tetsudou in Tsukioka area, Toyama City from 2003.

As a result of the upswing of the information technology in recent years, wide ranges of improvements of calling method of DRB have been appeared. These improvements can reduce the operation cost of DRB service, then in the future, we may be able to introduce DRB service much cheaper than before and number of DRB service may be increased.

2.2 External Effects by Bus Service and Related Studies Up to Now

Public transportation service is generally based on scale economy, which comes from that cost of one vehicle operation can be divided by many passengers sharing that service. Therefore, it becomes very difficult to provide service in the area with low density of demand. Furthermore, whenever the bus services are provided, the external negative effects such as increases of the stop time by getting on and off at bus stops and the running time to make detour by the calling in the DRB service may occur inevitably. Under such condition of the transport service market, the market equilibrium doesn’t become socially efficient. Therefore, we should internalize the external effect by adding the price of DRB, in order to lead the equilibrium closer to the social optimal situation.

Studies specifically taking up the external effect related with the bus service are limited to Suzuki (1987). Suzuki argued mathematically about the optimal arrangement of the bus stop which minimizes the amount of total boarding time of all users, aiming at the boarding increasing time due to the stopping at bus stops on route. When thinking only of the access time of the users, the larger number of bus stops concludes more desirable. However, decline of the driving speed, rise of the construction cost of bus stop facilities and increase of stopping time will occur. Therefore, Suzuki’s paper describes that it is important to carefully arrange a limited number bus stops. In case of the bus line from suburb to the downtown, because number
of passengers undergoing the influence by additional stop time increases as approaching the downtown, the paper shows that the external effect can be eased by making the bus stop interval longer as in the downtown.

2.3 Research Task of Bus Service in Urban Area and Concept of Our Study

With the deregulation of the bus service, the scrap and build in the bus lines are beginning with the suburbs. Therefore, it is necessary to review the influence of these logically and specifically. In order to cope with these problems facing bus service in the city, Nakamura (2002) has divided fields into 1) traffic control, 2) demand, 3) supply, 4) fiscal resource help system, and 5) facilities such as the bus terminal. Among these, the demand and the supply influence mutually each other and as the policy which changes the ideal way roughly, the reorganization of the network, the reconsideration of the way of operation and the fare rate system and the service of the guide information system and so on are given. It becomes the problem which is related to the degree of the uncertainties in the required time and the spread of the external effects. As the past research about the reorganization of the network, Nakagawa, D. et al. (1986) had studied the research of tactical approach to have considered the uncertainty of the public transportation. Also, Takayama, J. et al. (1997) had studied the research about the network reorganization introducing a high-speed bus service.

When considering the location form of the housing complex in suburb, the possibility that a detour type DRB becomes higher in the future. Therefore, this paper deals with the detour type DRB. Some researches about the typology of the demand response public transportation service have been accomplished (Dohi, T. et al., 2000, Kim, J. et al., 2002, Hirata, T. et al., 2003). These studies are focusing on the relationship between the characteristics of the users and the optimal bus operation type, but do not pay attention to the influence of the external effects which occur by the calling of DRB.

Timing of demand collection is one key issue for system design; if demand collection has already finished before the bus departure, operation schedule can be fixed, then the estimated pick-up time can be informed to the passengers. After the call back, all passengers need not suffer from unintended waiting, but they must obey the predetermined schedule. Alternate timing of demand collection is after the departure. Passengers call bus by pushing a button at a bus stop or call bus with other telecommunication tools. Demand call is transmitted to the bus driver, and then the bus goes around to the stops with passengers. This style seems suited for the area with middle density of demand, because passengers need not obey to the predetermined schedule. However, without beforehand information of real operation schedule, passengers must endure uncertain waiting time at bus stop and increase of on-vehicle duration due to detours. Because those disutilities are considered as external effects among passengers, free competition in market doesn’t give an efficient equilibrium, then some intervention such as fare control is needed.

In order to clarify the above characteristics of DRB system, this paper considers the combination of the detouring DRB which detours to each housing complex responding to the demand calls and the trunk line bus which runs only on the trunk road. We try to formulate a model which takes external effects of this bus system into account, and we show the optimal bus operation minimizing the total social cost which consists of user disutility and bus operating cost. Moreover, we clarify the influence of differentiating the fares of the DRB and the trunk line bus. In this paper, the DRB is assumed to be called by pushing a button at bus stop.
3. FORMULATION OF THE MODEL

3.1 Problem Settings of Our Study

This paper assumes a hypothetical urban form shown in Figure 1 and it analyzes the DRB operation problem under the following suppositions.

1. Along the trunk road which links a bus depot in the suburb, and the downtown, $n$ housing complexes exist. Housing complex $i$ is locating $l_i$ (km) aside from the $i$th intersection on the trunk road, which is $L_i$ (km) from the previous intersection for the housing complex $(i - 1)$ on the trunk road.
2. Between the bus depot in the suburb and the downtown, the trunk line bus ($M$) and the demand responsive bus ($D$) are operated. Operation of both trunk line bus and DRB is provided in the predetermined schedule, with intervals $I^M$, $I^D$ (h/veh), respectively. It neglects any delays due to traffic congestions, and all operations are done with speed of $v_b$ (km/h) irrespective of the kind of the bus.
3. For trunk line users, walking access from the housing complex core to the nearest bus stop on the trunk line yields disutility. Walking speed is set as constant $v_w$ (km/h). Bus users arrive at the bus stop at the estimated time of bus arrival. Stopping time at bus stops is ignored.
4. DRB users call a DRB by pushing a button at the bus stop, just after the arrival to the bus stop. At the bus stop, time table is displayed based on the earliest arrival time when each bus comes without any detours to the upstream housing complexes. Actual bus service then delays from that schedule if that DRB is called.
5. Number of passengers (demand) at $i$th housing complex is given as the rate per unit hour as $X_i$ (person/h). All passengers are homogenous and go to the downtown.
6. Vehicle capacity constraint or internal congestion in buses is ignored. Then, all users can ride on the bus which they want to ride on.

3.2 Formulation of Disutility of Trunk Line Bus

The user compares the utilities of the trunk line bus and that of DRB and chooses either. In this problem, all service level variables have negative effects, then, we formulate the model using disutilities, instead of utilities.

Disutility of trunk line bus user from the $i$th housing complex, $f^M_i$ is defined as the sum of the disutility by expected schedule cost for the waiting time in house which is caused by that there is no bus service on the desired time, the disutility by the length of the walking time to the trunk
line bus stop, and the disutility which depends on the boarding time to the downtown after ride and the fare.

\[ f_i^M = a \frac{I^M}{2} + b \frac{l_i}{v_w} + c \sum_{j=1}^{i} \frac{L_i}{v_b} + \text{Fare}_i^M \] (1)

where, \( I^M \): trunk line bus service interval (h/veh), \( a \): time value for schedule mismatch in the house (yen/h), \( b \): walking time value (yen/h), \( c \): time value of the required on board time (yen/h), \( v_b \): bus speed (km/h), \( v_w \): walking speed (km/h), \( L_i \): interval of trunk line bus stop \((i - 1)\) and \(i\) (km), \( l_i \): distance between housing complex \(i\) and the trunk line bus stop (km), and \( \text{Fare}_i^M \): trunk line bus fare of the user in housing complex \(i\) to the downtown (yen).

### 3.3 Formulation of Disutility of Demand Responsive Bus

It defines the disutility \( f_i^D \) when the user in housing complex \(i\) uses a DRB as follows.

\[ f_i^D = a \frac{I^D}{2} + d \left( \sum_{k=i+1}^{n} \frac{2\delta_k l_k}{v_b} + \frac{l_i}{v_b} \right) + c \left( \sum_{j=1}^{i} \frac{L_j}{v_b} + \sum_{j=1}^{i-1} \frac{2\delta_j l_j}{v_b} + \frac{l_i}{v_b} \right) + \text{Fare}_i^D \] (2)

where, \( I^D \): interval of the DRB service (h/veh), \( d \): time value of waiting time in the bus stop (yen/h), \( \text{Fare}_i^D \): fare of the DRB of the user in housing complex \(i\) (yen), \( \delta_i \): detour probability of one DRB at housing complex \(i\).

The first term of the right side shows an expected schedule cost for the waiting time in house which is caused by there is no bus in the desired time. The second term of the right side shows a disutility for waiting time in the bus stop which is caused by the delay time which the DRB detours the upstream side housing complex \(k \in [i + 1, n]\) and the time from the trunk road to the bus stop in the \(i\)th housing complex. The third term of the right side shows the duration to the downtown which consists of drive time on the trunk road after ride and the required time due to the detours round to the downstream side housing complex \(j \in [1, i - 1]\). The forth term of the right side is the fare.

Because a bus goes around when it is called from the user in the other housing complex, the user cannot grasp the disutility of the DRB definitely. When supposing that the occurrence of the DRB users follows a Poisson process, DRB detour probability in housing complex \(i\) is shown by the following equation.

\[ \delta_i = 1 - \exp(-\sigma_i X_i I^D) \] (3)

where, \( \sigma_i \): DRB choice probability of the user in housing complex \(i\). A DRB choice probability is given by the following the binary logit model using the user disutilities of each bus as shown above.

\[ \sigma_i = \frac{1}{1 + \exp[\alpha(f_i^D - f_i^M)]]} \] (4)

where, \( \alpha \): a parameter describing the elasticity for the monetary unit of disutility difference.

The external effect \( E_i \) on DRB due to the call by a user in housing complex \(i\) is given as the following.

\[ E_i = \left( 2c \frac{l_i}{v_b} \sum_{j=i+1}^{n} \sigma_j X_j I^D + 2d \frac{l_i}{v_b} \sum_{k=1}^{i-1} \sigma_k X_k I^D \right) / (\sigma_i X_i I^D) \] (5)
where, the first term of the right side is the increase of required on-vehicle time for the passengers already getting on at the outer bus stops. The second term is the increment of disutility for the waiting time of the later passengers at the inner bus stops. \( \sigma_i X_i I^D \) is the number of users per one DRB in the housing complex \( i \). The external effect should be internalized if different fares are set for DRB and the trunk line bus, \( \Delta \text{Fare}_i (= \text{Fare}_i^D - \text{Fare}_i^M) \).

### 3.4 Formulation of Bus Operation Cost

Operation cost per bus is considered to be proportional to the required time for operation. For DRB, we consider the binding hours of a driver and a vehicle, based on the most costly case when that DRT is called at all housing complexes and directly return to the depot in suburbs. Total operation costs per unit of time of the trunk line bus and the demand responsive bus are \( g^M, g^D \), defined as follows, respectively.

\[
g^M = (MOT) \left( \frac{\sum_{j=1}^{n+1} \frac{L_j}{v_b}}{I_M} \right) \frac{1}{I^M} \quad (6)
\]
\[
g^D = (DOT) \left\{ \sum_{j=1}^{n} \left( \frac{L_j + 2l_j}{v_b} \right) + \frac{L_{n+1}}{v_b} \right\} \frac{1}{I^D} \quad (7)
\]

where, \( MOT \): operation cost of the trunk line bus per unit of hour (yen/h/veh), \( DOT \): operation cost of the DRB per unit of hour (yen/h/veh).

### 3.5 Formulation of the Total Social Cost

The fare is canceled in the whole society because it is nothing more than money transactions internal for the social point of view. Therefore, the total social cost \( TC \) (yen/h) is defined as follows;

\[
TC = \sum_{i=1}^{n} \left( \left( f_i^M - \text{Fare}_i^M \right) (1 - \sigma_i) X_i + \left( f_i^D - \text{Fare}_i^D \right) \sigma_i X_i \right) + \left( g^M + g^D \right) \quad (8)
\]

\( TC \) does not include the fares, but due to the definitions of \( f_i^M \) and \( f_i^D \), we must subtract the fares in order to get the social cost.

### 4. TOTAL SOCIAL COST MINIMIZATION

From the social point of view, it is desirable to minimize the total social cost \( TC \), formulated as follows;

\[
\min_{I^M, I^D, \text{Fare}_i^M, \text{Fare}_i^D} TC \quad (9a)
\]

s.t.

\[
\left( \frac{2 \sum_{i=1}^{n+1} L_i}{v_b} \right) / I^M + \left( \frac{2 \sum_{i=1}^{n+1} L_i + \sum_{i=1}^{n} 2l_i}{v_b} \right) / I^D = \text{const.} \quad (9b)
\]

\[
I^M \geq 0, \quad I^D \geq 0 \quad (9c)
\]

Eq. (9b) is a constraint about the total number of buses required for providing the two type bus service. Because we consider one way inbound service in the morning hours, all buses directly return to the bus depot in the suburb without any service, after finishing the inbound operation.
from the suburb to the downtown. For the DRB, similar to the operation cost account in eq.(7), longest binding time is considered, including detours to all housing complexes. Eq. (9c) are non-negative conditions for the service interval of both type buses.

Because this mathematical problem has very non-linear objective function of $TC$, the optimal solution is not easy to be analyzed. Therefore, several iterative computations are required as the following procedure; at first, difference of the fares between DRB and the trunk line buses $\Delta\text{Fare}_i$, and bus operation intervals, $I^D$ and $I^M$ are assumed. Next, an initial value of DRT detour probabilities $\delta_i$ for every housing complex are set. Then, in order to get value of RDB choice probability $\sigma_i$, we calculate eq.(4) and get value of RDB detour probability again through eq.(3). Such iterative process were repeated until we get $\sigma_i$ and $\delta_i$, that satisfy both eq.(3) and eq.(4). After these calculations, we evaluate the total social cost $TC$ through eq.(8) for the given values of $\Delta\text{Fare}$, $I^D$ and $I^M$. At last, comparison of these several solutions, we can numerically obtain the optimal combination of $\Delta\text{Fare}, I^D$ and $I^M$.

5. NUMERICAL EXAMPLE FOR OPTIMAL BUS OPERATION

To simplify the problem, bus stop intervals along the trunk line $L_i$, intervals between each housing complex and the bus stop along the trunk line $l_i$ and the number of passengers $X_i$ are set constant, as described by $L$, $l$, $X$, respectively. In this numerical example, we set parameters and the above constants as follows; the time value for schedule mismatch in the house $a = 300$ (yen/h), the walking time value $b = 2,000$ (yen/h), the time value of required on board time $c = 400$(yen/h), the time value of waiting time in the bus stop $d = 600$ (yen/h), the operating cost of the trunk line bus $MOT = 3,500$ (yen/h/veh), the operating cost of the DRB $DOT = 2,800$ (yen/h/veh), the demand $X = 10$(person/h), the bus speed $v_b = 15.0$ (km/h), the walking speed $v_w = 3.0$ (km/h), interval of the trunk line bus stops $L = 3.0$(km), the distance between housing complex and the trunk line bus stop $l = 0.6$ (km), the number of the housing complexes $n = 10$, the elasticity parameter in the logit model $\alpha = 0.002$, and 20 vehicles of total bus number.

Because the nominal fare levels do not affect on the choice of the users except the difference of fares between the trunk line bus and the DRB $\Delta\text{Fare}_i$, and total collected fare does not affect on the social cost, we can take $\Delta\text{Fare}_i$ as a policy variable regardless of the sum of the external effects. For the simplicity, we further consider the $\Delta\text{Fare}_i$ constant regardless of housing complex.

5.1 Comparison of Total Social Cost by the Fare Difference

For given value the fare difference, $\Delta\text{Fare}$ of the demand responsive bus and the trunk line bus, iterative calculation is done in order to seek the operation interval of two type buses to minimize the total social cost $TC$. The change of the minimized total social cost for the different value of the fare difference $\Delta\text{Fare}$ is shown in Figure 2. When the difference is 1,000 yen, the total social cost becomes minimal.

The computed total social cost, operation intervals and operation cost are shown in Table 1 for the two cases; fare difference is 0 and 1,000 yen. With the introduction of fare difference of 1,000 yen, the total social cost becomes 600 yen smaller than the case of no fare difference. Demand responsive bus choice probability $\sigma_i$ at each housing complex for those two cases is shown in Table 2 and demand responsive bus detour probability $\delta_i$ is shown in Table 3. In case of no fare difference the total number of the DRB users is 54 (person/h), and it decreases as small as 15 (person/h) by introducing the extra fare of 1,000 yen. This decrease of users further lessens the substantial decrease of the probability of DRB detours.

Figure 3 shows the change of average disutility for both type buses along the change of the fare difference. The introduction of the fare difference improves both users, as shown in the down-
ward curves in Figure 3. The increase of trunk line bus users enables more frequent operation and shorter waiting time for the users. On the other hand, the decrease of DRB users resulted less frequent DRB operation and longer waiting time at home, but also the remarkable decrease of disutilities owing to the smaller DRB detour probabilities. Despite of improvements of both bus service, those disutilities does not cross each other, then DRB always offers better option than the trunk line bus. Therefore, too large fare difference has negative effect by compulsory attribute more users for the inferior option. This is why, we observes an optimal point around 1,000 yen for total social cost minimization.

In order to scrutinize the effect of DRB system, we also calculate the total social cost $TC$ for the case where all buses are solely operated on the trunk line. The result is 122,725 yen, 240 yen larger than the combination system without fare difference. We can conclude that introduction of DRB service, which is more closer to the houses, improves the transportation service, and
5.2 External Effect and the Fare Difference

The value of the external effects from a calling at each housing complex when the fare difference is 1,000 yen is shown in Table 4. The value of the fare difference is much bigger than the estimated external effects for every housing complex, so the external effects are proved to be fully internalized by the fare difference. It is also found that external effect does not much differ across the housing complexes, then we need not set the fare difference $\Delta Fare_i$ individually for each complex.

6. CONCLUSION

Demand Responsive Bus system has gathered attentions so far because of the positive effect from the flexible operation forms, but inevitable externality from calling has not yet analyzed. This paper discusses the externality in the DRB system combined with the ordinal bus service with predetermined schedule in suburb area, and formulated a mathematical problem minimizing the total social cost.

As a result of the numerical calculations, to combine a trunk line bus and DRB service gives a solution less socially expensive than the conventional trunk line system with predetermined schedule. Introduction of fare difference avoiding the overuse of DRB by the users neglecting the external effects is proved to further improve the efficiency, and a flat fare difference for
all housing complexes is enough to internalize the social externalities generated by the calling DRB at each bus stop.

These results show the value of DRB in suburb area and importance of consideration of externality. Due to the non-linearity of the problem, we have much relied on the numerical calculations rather than theoretical analysis, then the derived values strongly depend on the given parameter values. We should seek more theoretical consideration in the future in order to derive more robust findings. In this model, we consider the unidirectional service from the suburb to the downtown, but we should expand the model applicable for the bi-directional service including the service from the downtown to the suburb.

REFERENCES

a) Books and Books chapters

b) Journal papers

c) Papers presented to conferences

d) Other documents
Nomura Research Institute (1976) The research of the urban traffic system which is based on the service improvement of the public transportation. (in Japanese)