A RANK SIZE RULE IN A FIRM, PRODUCED FROM A HIERARCHICAL BRANCH OFFICE LOCATION MODEL

Makoto Okumura,
Center for Northeast Asian Studies, Tohoku University, 41 Kawauchi, Aoba-ku, Sendai, 980-8576, Japan

Makoto Tsukai
Department of Civil and Environmental Engineering, Hiroshima University, 1-4-1 Kagamiyama, Higashi-Hiroshima, 739-8527, Japan

and

Naoki Takada
Fujioka Public Work Office, Gumma Prefecture, 124-5, Shimokurisu, Fujioka City, Gumma, 375-0014, Japan

Although many studies of the rank-size rule have examined cities, few have provided a micro-behavioral foundation for the rule itself. Using a hierarchical branch office location model, this study confirmed that the rank-size rule appears in the hierarchical structure of a firm calculated using the model. Results show that the power coefficient does not change through a numerical decrease in branch offices when the branch set-up cost increases. Results also show, however, that the power coefficient becomes larger and that employment becomes more evenly distributed when the effectiveness of branch offices, in terms of the ability to compress and aggregate business information, is enhanced. Progress in information and communications technology (ICT) has enhanced the effectiveness of branch offices. Therefore, a flatter organization of firms can be expected in the future.

1. Introduction

1.1 Rank size rule and national planning

The rank-size rule is an empirical proposition that the distribution of the population scale and the scale ranking of cities produce a straight line when both are shown on a logarithmic scale. This empirical finding was first pointed out by Auerbach in 1913 and formulated by Zipf...
as early as 1949. Assessments based on statistical analyses are continuing, even in recent years (e.g. Overman and Ioannides, 2001).

Presuming that the scale ranking is $y$, and presuming that the population scale of the city is $x$, then a rank-size rule can be represented as follows.

$$\log y = \log A - \alpha \log x,$$

In that equation $\alpha$ is called a power coefficient representing the concentration of the city; $A$ is a constant to be regressed. A small power coefficient $\alpha$ reflects that a population is accumulated in a few cities; a large coefficient signifies a dispersed population among many cities.

The monopolistic concentration of population and economical activities on the primal metropolitan area (frequently the national capital), and concurrent out-migration from and decline of remotely located rural areas have become important policy issues in many countries. If the rank-size rule were strictly true, then the feasibility of national development policies aimed at development of rural cities might be problematic because, by the rank-size rule, the number of cities possessing population greater than a certain threshold is already fixed. The development policy might become no different to a children’s game of musical chairs.

Hatta (2006) pointed out that the main cause of the monopolistic concentration in Tokyo around 1990 was the company’s restructuring of central administrative functions from Osaka, corresponding to the travel time reduction by the rapid train service of Japan’s bullet trains (Shinkansen). The current authors agree that the main cause of nationwide population distribution is a company’s business organization design. Especially, the present development of information and communication technology (ICT) might be enhanced further, which might alter the corporate organization. The kind of change that will be made in the rank-size rule of the city system is an important issue related to national land policy.

1.2 Existing studies of causes of the rank-size rule

Numerous empirical studies of the rank-size rule in the city distribution have been conducted over many years, but no stylized theory yet exists to explain why the rank-size rule has come to prevail.

Simon (1955) proposed a virtual dynamic process of city birth and growth: once a virtual new city of fixed population is founded, that city is considered to be either absorbed by an existing city with probability proportional to the population of the existing city, or left alone as an independent city. Simon showed that the population distribution of the cities follows a rank-size rule when the probability of the birth rate of new city approaches zero. Krugman (1996) criticized the assumption of the zero birth rate in this model. Duranton (2006) provided a micro-foundation and an economic foundation to the Simon’s model, but he based it strongly on the unrealistic assumption that all products of all cities have equal quality.

Beckmann (1958) is another pioneering work for the urban rank-size rule based on employment size, as determined by the complementary area size, followed by improved notification by Beckmann and McPherson (1970). They ignore geographic conditions but assume a constant number of lower layer cities connected to an upper layer city.

Gabaix (1999) showed that a rank-size rule prevails in a city system under Gibrat’s empirical rule of growth: cities of similar scale have similar proportional growth percentages. Ioannides

Regarded from the viewpoint of city systems, the rank-size rule appears in Christaller’s city system. However, central place theory, including the work of Christaller, has tenuous economic foundations and is frequently criticized as “mere geometry”: only a drawing of a picture reflecting a fact. Fujita et al. (2001) simulated an economic geography model and showed the appearance of Christaller-like city system structure, but the simulations fail to replicate the rank-size rule.

1.3 Business organization of companies and the rank size rule

Pred (1976) described the importance of corporate organization as a cause of structural changes in the city system. Recently, Mizuta (2008) aggregated the number of e-mail transmission logs in a company and analyzed them in relation to the business organization of the company. He found that the e-mail transmission among business sections agrees with the hierarchical personnel organization structure with the president at the top. Moreover, the numbers of e-mails sent to and sent from each section satisfy a rank-size rule. Numbers of the rank-size rule observed in e-mail messages might also apply in cases of in-house employment distribution in a company if the transmitted numbers of e-mails per worker differ little.

Figure 1 is a graph portraying the area employment distributions of the top three megabanks in Japan based on the company’s annual reports for investors. It can be confirmed that those employment distributions approximately follow a rank-size rule.

1.4 Purpose of this research

This research was undertaken to build a model to describe the formation of a hierarchical structure in business organization from a microscopic viewpoint. A cost-minimization rule will produce a rank-size rule in the employment distribution of a firm. We do not assume any dynamic process at the city level as Simon (1955) does. Instead, based on a static model in a company, a comparative statics approach is used to analyze the plausible effects of future ICT development on the power coefficient of the rank-size rule in the company’s employment distribution.
2. Hierarchical branch office location model

2.1 Existing research related to hierarchical branch office location models

Some studies (Hino, 1999; Suda, 1998) have been undertaken to model the branch office location of a company and to compare calculated results with the actual distributions of branch offices. The authors of this paper have also worked with such branch office location models, applying one model to assess the regional effects of intercity transportation development projects (Tsukai and Okumura, 2003), and expanding the model by considering uncertain fluctuation of jobs (Okumura and Tsukai, 2008). Nevertheless, these treatments of models address only the problem of whether branch offices at one intermediate layer should be inserted between customers and the headquarters office or not. They are insufficient to describe a multilayer branch office structure permitting a situation in which an upper-layer branch office manages several substratum branches.

On the other hand, Sahin and Sural (2007) classified studies of multilayer facility location models in electric communication network research, showing that the number of hierarchies is predetermined in all those studies. A logistic center location model described by Kijimanawat and Ieda (2004) is an exceptional study that determines the number of hierarchies endogenously; nevertheless, their model cannot describe a tree structure with a headquarters at its root.

2.2 Business organization structure of a representative company

Unrelated to previous branch office location models, this study proposes a model in which both the number of hierarchies and the locations of branch offices in an organization network rooted by a headquarters are decided endogenously.

We consider a representative company in the business service industry. Hereinafter, we explain the model assumptions.

1 This company provides business services to all customers distributed spatially throughout the entire country. The company comprises one headquarters office and several branch offices on different layers.
2 Branch offices on the first (bottom) layer control only direct communications with customers. Branch offices (and the headquarters office) on the upper layers manage the linked substratum branch offices; they can communicate directly with customers residing near the office.
3 They exchange business information through face-to-face contact between the front office and the customers, and between linked offices. Such an information exchange requires certain transportation costs reflecting the distance between locations.
4 A branch office takes the role of compressing and aggregating the business information gathered from the substratum branch offices or from customers. In other words, the branch office screens and only passes the difficult information through to the upper layer, that are included in the gathered information with a fixed proportion. We define that percentage of information to be sent the upper layer as the “information concentration ratio”, as indicated by $R$.
5 Corresponding to the business information quantity which each branch and headquarters office handle, employment costs are necessary.
Figure 2. Organization structure of a representative company.

Figure 3. Calculation procedure of multi-layer hierarchies by the two-layer location model.

The company tree structure, including one headquarters office and branch offices is shown in Figure 2.

2.3 Computation procedure

The multilayer hierarchical branch office location model is based on the two-layer branch location model presented by Okumura and Tsukai (2008). The model is used repeatedly from the bottom-most layer up to the headquarters. The two-layer branch location model facilitates the judgment of whether to locate an upper-layer branch office to economize communications between the substratum branch office and the headquarters office or not. The computation procedure is depicted in Figure 3.

1 Initially, as the first iteration, a two-layer location model is applied with consideration of each customer as a substratum branch office to judge the need of the first layer branch offices.

2 In the $n$th iteration, we multiply $R$ with information from branch offices located at the previous $n-1$th iteration, and consider them as the substratum branch offices at the $n$th iteration. For branch offices (or customer) already managed using a direct link to the headquarters in the previous $n-1$th iteration, a dummy branch substratum office is placed. Following these preparations, the two-layer location model is applied.
3 Such a procedure will be iterated. Through the iterations, the cost reduction effect of a new branch office becomes smaller. Finally, no additional location of a new branch is sufficiently profitable to cover the set-up cost. The iteration process stops at that point.

2.4 Formulation of the two layers branch office location model

This two-layer location model is formulated to determine the additional insertion of upper branch offices yielding total cost reduction through the compression of business information exchanges between the headquarters and the substratum branch offices located in the \( n-1 \)th iteration.

2.4.1 Location cost of the headquarters office

When the employment size of the headquarters office is large, a wider space must be prepared. Then, the location cost of the headquarters office: \( C_{n0}^{n} \) is given as

\[
C_{n0}^{n} = f_0 + (h_0 + p_0)s_{n0}^{n},
\]

(2)

where \( f_0 \) stands for the fixed set-up cost of the headquarters office, \( h_0 \) represents the wage per employee at the headquarters, \( p_0 \) signifies floor space rent per employee, and \( s_{n0}^{n} \) denotes employment at the headquarters in the \( n \)th iteration.

2.4.2 Location cost of branch offices

Similarly, the location cost for the upper layer branch offices, located in the \( n \)th iteration \( C_{n1}^{n} \) is given as

\[
C_{n1}^{n} = \sum_{j=1}^{J} \{ f X_{nj}^{n} + (h_j + p_j)s_{nj}^{n} \},
\]

(3)

where \( f \) represents the fixed set-up cost of a branch office, \( h_j \) is the wage per employee at the branch office in city \( j \), \( p_j \) denotes the floor space rent in city \( j \) per employee, and \( s_{nj}^{n} \) represents employment at the branch office in city \( j \).

2.4.3 Controllable variables

\( X_{nj}^{n} \) is a 0–1 controllable variable indicating the location of the upper layer branch office in city \( j \):

\[
X_{nj}^{n} \in \{0, 1\} \forall j
\]

(4)

The number of employees at branch office \( s_{nj}^{n} \) is determined such that it meets the needs from the linked substrata branches \( i \). It satisfies the following:
where $\lambda^n_i$ denotes the total business information quantity from the substratum branch office in city $i$. We introduce a control variable $Y^n_{ij}$, indicating the quota of the information heading for upper branch $j$ from the lower branch office $i$. It is limited by the existence of upper branch office $X^n_j$ in city $j$ as follows.

$$0 \leq Y^n_{ij} \leq X^n_j \quad \forall i, j$$

(6)

Accordingly, when $X^n_j = 0$, both $s^n_j$ and $C^n_4$ must be zero.

### 2.4.4 Information exchange cost

Next, let us consider the information exchange cost between the linked upper and lower branch offices. We define the unit of business based on the volume requiring one unit of information exchange. Then the total information exchange cost $C^n_2$ can be given as follows.

$$C^n_2 = \sum_{j=1}^{J} \sum_{i=1}^{I} \lambda^n_i d_{ij} Y^n_{ij},$$

(7)

In that equation, $d_{ij}$ stands for the information exchange cost between upper and lower branch offices for one employee, given exogenously according to the transportation conditions.

Business information collected at the upper branch office is concentrated with the rate of $R$; then it is exchanged further with the headquarters office. The total information exchange cost between the upper branches and the headquarters is given as

$$C^n_3 = R \sum_{j=1}^{J} s^n_j d_{j0},$$

(8)

where $R$ signifies the information concentration ratio. Also, $d_{j0}$ stands for the information exchange cost between upper branch and headquarters offices for one employee. Its value is assigned exogenously according to the transportation conditions.

Direct information exchanges between the lower branch and the headquarters become less expensive than locating the additional upper branch office in middle if the lower branch office location is not far from the headquarters location. In that case, all the business exchange from lower branch $i$ occurs directly to the headquarters office. The total cost of such direct exchanges $C^n_4$ is given as shown below.

$$C^n_4 = \sum_{i=1}^{I} \lambda^n_i d_{i0} Z^n_{i0},$$

(9)
In that equation, $d_{i0}$ represents the direct information exchange cost between a lower branch and the headquarters office for one employee of the lower branch office, and $Z_{i0}^n$ is a controllable variable representing the share of headquarters exchange from the lower branch office in city $i$.

$$0 \leq Z_{i0}^n \leq 1 \quad \forall i$$

(10)

2.4.5 Cost minimization problem

The company is considered to minimize the total cost $C_T^n$ composed of the cost components stated above; the problem is formulated as presented below.

$$\min_{X_j^n, Y_{ij}^n, Z_{i0}^n, s_j^n} C_T^n = C_0^n + C_1^n + C_2^n + C_3^n + C_4^n$$

(11)

s.t. $X_j^n \in \{0, 1\} \quad \forall j$

(12)

$$\sum_{j=1}^J (Y_{ij}^n + Z_{i0}^n) \geq 1 \quad \forall i$$

(13)

$$0 \leq Y_{ij}^n \leq X_j^n \quad \forall i, j$$

(14)

$$0 \leq Z_{i0}^n \leq 1 \quad \forall i$$

(15)

$$s_j^n = \sum_{i=1}^I \lambda_i^n Y_{ij}^n \quad \forall i, j$$

(16)

$$s_0^n = \sum_{i=1}^I \lambda_i^n Z_{i0}^n + R_C \sum_{j=1}^J s_j^n \quad \forall i, j$$

(17)

Of those expressions, (13) shows the constraint condition in each lower branch office; intractably difficult work is fully covered under the control either of an upper branch office or the headquarters office. Equation (17) shows that the total employment at the headquarters must satisfy demand both from upper branch offices and lower branch offices.

2.5 Resultant employment distribution

After the iterated computations, the total numbers of employees at branch offices and headquarters are given respectively as follows.

$$S_j = \sum_{n=1}^N s_j^n \quad \forall j.$$  

(18)

$$S_0 = s_0^N,$$  

(19)
In those equations, \( S_j \) is total number of employees at branch office in city \( j \); \( S_0 \) represents the total number of employees at the headquarters office.

### 3. Analysis of Japanese company organization

#### 3.1 Setting of a representative company and parameters

We consider a representative company having demand of service equivalent to 1,000 employees at the bottom front facing to the customers, from all over Japan. The spatial distribution of the customers is set proportionally to the general size of economic activities, which is given as the total number of employment for all industries in each of 194 zones in 2004, as depicted in Figure 4. Location of the headquarters office is fixed as downtown Tokyo (Tokyo 23 Wards).

Each branch office worker uses 15.2 m\(^2\) of floor space, irrespective of the geographical location. Land rent is set according to the empirical land-rent data for the city, which has the largest number of employees within each zone, as portrayed in Figure 5. In contrast to the land rent, we neglect the spatial difference of workers’ wage rates and set the value as 2,960,000 yen/year, irrespective of the location. Interzonal transportation costs are inferred from a comparison of generalized cost including time of the railway shortest time route and that of the rail–air shortest time route. The time value of one hour is assumed as 3,000 yen.
Figure 5. Floor rent for one office worker in 194 Zones.

Table 1. Number of the located offices

<table>
<thead>
<tr>
<th>Fixed location cost $f$ (10000yen)</th>
<th>Information Concentrate Ratio $R$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td>2,000</td>
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<td>7,000</td>
<td>13</td>
</tr>
<tr>
<td>8,000</td>
<td>11</td>
</tr>
</tbody>
</table>

3.2 Computation results of the number of branch offices and organization structures

The remaining exogenous parameters are the information concentration ratio $R$, fixed set-up cost of the headquarters office $f_0$, and the fixed set-up cost of branch office $f$. To simplify the analysis using two parameters, we assume that $f_0 = f$. Because we fix the location of the headquarters as Tokyo, parameter $f_0$ does not affect the result.

For several combinations of those two parameters of $(R, f)$, the multilayer hierarchical location model is applied to determine the number and locations of branch offices. Table 1 presents the number of the offices located in Japan including both the headquarters and branches.
Either the increase of fixed set-up cost \( f \) or the increase of the information concentration ratio \( R \) gives the result of fewer branch offices. The same number of branch offices yields several different combinations of the two parameters, but the actual geography of the organization structure differs. For example, we show that the organization structures containing 11 offices, obtained respectively from \((f, R) = (3000, 0.6)\) and \((f, R) = (5000, 0.5)\) in Figures 6 and 7. In those figures, 194 zones are arranged in the horizontal axis in north–south order; the total employment at each office is shown on the vertical axis. The superscript number on each branch location shows the hierarchy, obtained as the largest iteration number in the calculation when that location keeps a branch office. Straight lines in the figure show management linkages.

When we compare the figures, the case of smaller fixed set-up cost and larger information concentration ratio (Figure 6) fosters larger employment at the Tokyo headquarters office but fewer employees at the branch office in Osaka. On the other hand, Himeji, Hiroshima, and Kumamoto are arranged respectively in the substratum under Osaka, Okayama, and Fukuoka, where the hierarchy number is 2. In Figure 7, in the case of expensive fixed set-up costs and more efficient branch office technologies, branches in Himeji and Kumamoto disappear and the hierarchy number of Osaka and Fukuoka become one. The resultant organization structure seems flatter.
4. Rank size rule for employment

4.1 Comparison of the inclinations

Concerning on the several combinations of \((f, R)\) having 11 locations of branch and headquarter offices, we draw the rank size relationships both on logarithm axes, as Figure 8. In this figure, we found larger value of the power coefficient in the rank size rule, showing the steeper inclination of the graph, when fixed cost is larger and information concentration is stronger.

4.2 Sensitivity analysis for the information concentrate ratio

We maintain a fixed set-up cost parameter \(f\) equal to 2000. Then we draw rank-size relations in employment for several values of the information concentration ratio \(R\), as shown in Figure 9. A small concentration ratio signifies that the concentration function of the branch office is strong; the power coefficient is large and the inclination of the graph is steep. This is true because a smaller amount of information is sent upward, the necessary employment at upper branches and the headquarters decreases.

4.3 Sensitivity analysis for the fixed set-up cost

We change the value of the fixed set-up cost \(f\) while maintaining the information concentration ratio constant as 0.4. Figure 10 presents rank-size relations in employment for several
values of the fixed cost parameter. No changes are observed in the inclination of the graphs, although the numbers of branch offices might differ.

4.4 Change of corporate organization

Next we confirm the changes in the corporate organization structures because of the changes of parameter values. Figure 11 portrays the case of \((f, R) = (3000, 0.5)\), which is different in
terms of the fixed set-up cost $f$ from the case presented in Figure 7. Branch offices of the hierarchical one newly appeared in Nagoya, Osaka, and Fukuoka, but a new branch office does not appear in the upper layer. In contrast, Figure 12 depicts the case of $(f, R) = (5000, 0.2)$, which differs in $R$ from the case shown in Figure 7. In this case, the upper branch office at Sendai appears over the Morioka branch. Simultaneously, a new branch office is inserted at Niigata to manage 38, 44, 74–77, and 79 zones, which had been managed directly by the headquarters office in Tokyo. In that way, the change of the power coefficient in rank-size rule corresponds to organization changes other than the first bottom layer.

5. Conclusion

This study proposed a multilayer hierarchical branch office location model and showed that the rank-size rule appears in the employment in hierarchical organization of a company. Sensitivity analysis was done for the parameters of fixed set-up cost and the information concentration ratio.

Results show that if the concentration function becomes stronger in branch office because of development of ICT (smaller $R$ value) in the future, then the power coefficient becomes larger, engendering the appearance of a flatter business organization. However, a different scenario of ICT development presents a qualitative change of the exchanged information: the branch office
relied more frequently on the upper organization. That might increase the $R$ value, which is the opposite exogenous change from that demonstrated in our analysis above.

Organizational changes analyzed here might further alter the demand for intercity transportation service. Such effects will expand further according to the macroscopic changes of systems of the cities themselves. We must continue to analyze such causes and attempt to provide a theoretical foundation for recent observations that the power coefficient seems larger in developed countries, as reported by Soo (2005) and Rossi-Hansberg and Wright (2007).

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Send correspondence to Makoto Okumura: mokmr@m.tohoku.ac.jp

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